



Autumn Block 1

Ratio and proportion

Foundation Tier

When working with ratios, look carefully for which value(s) you've been given. Label each number carefully, so you don't get mixed up.

A
Mo and Ron share money in the ratio 3 : 5
Ron receives £50
How much does Mo receive?

Mo : Ron
3 : 5
30 : 50
× 10

Mo receives £30.

B
Mo and Ron share money in the ratio 3 : 5
Ron receives £50 more than Mo.
How much does Mo receive?

2 parts = 50
1 part = 25
÷ 2

Mo : Ron
3 : 5
75 : 125
× 25

Mo receives £75.

In **direct proportion** relationships, both quantities are multiplied or divided by the same value.

$\times 10$ £1 = £1,800
 $\div 2$ £10 = £18,000
 $\div 2$ £5 = £9,000

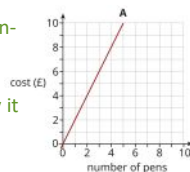
If we multiply the value in pounds, we do the same to the value in pence.

$\div 4$ Marking 4 books takes 40 minutes
 $\times 20$ Marking 1 book takes 10 minutes
 Marking 20 books takes 200 minutes

Marking more books takes more time.
Marking less books takes less time.

The more pens we buy, the more money it will cost (multiply both).

The less pens we buy, the less money it will cost (divide both).

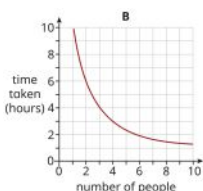


In **inverse proportion** relationships, we do the inverse operation to each value e.g. multiply one and divide the other.

$\div 3$ 3 cleaners take 90 minutes
 $\times 10$ 1 cleaner takes 270 minutes
 10 cleaners take 27 minutes

Less cleaners take more time to clean a house.

More cleaners take less time to clean a house.



The more people we have, the less time the job takes.

The less people we have, the more time the job takes.

Step 1 Solve problems with ratio

Step 2 Direct proportion

Step 3 Best buy problems

Step 4 Conversion graphs

Step 5 Exchange rates

Step 6 Inverse proportion

Step 7 Direct proportion equations (E)

Step 8 Direct and inverse proportion graphs (E)

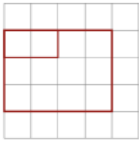
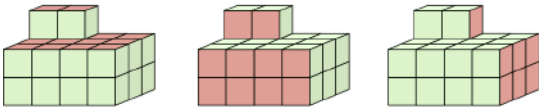
Key vocabulary

ratio	comparison of two or more values
share	divide a quantity into equal-sized groups or parts
equal parts	parts of a whole with the same value
whole	total or complete amount
directly proportional	relationship between two quantities where they increase or decrease at the same rate
scaling	multiplication of each value by a constant to proportionally increase or decrease the size
unit price	cost of a single unit of a product, found by dividing the total price by the number of units
conversion	process of changing a quantity from one unit or form to another
origin	where the axes of a graph intersect, at coordinates (0,0)
linear	relationship or pattern that changes at a constant rate, forming a straight line when graphed
exchange rate	value at which one currency can be exchanged for another
currency	system of money used in a particular country
convert	change a quantity from one unit or form to another
inversely proportional	relationship between two quantities where, as one quantity increases at a rate, the other quantity decreases at the same rate
assumption	something accepted as true without proof

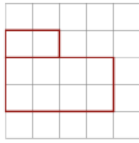


Autumn Block 2

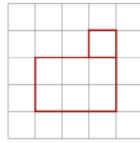
Area and volume Foundation Tier



Plan



Front elevation



Side elevation

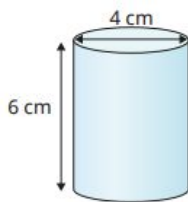
(view from above) (view from the front) (view from the side)

$$V = \pi r^2 h$$

$$h = 6\text{cm}$$

$$D = 4\text{cm so } r = 2\text{cm}$$

$$V = \pi \times 2^2 \times 6 = 75.4\text{cm}^3$$

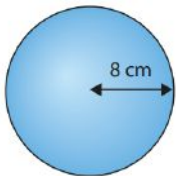


$$V = \frac{4}{3} \pi r^3$$

$$r = 8\text{cm}$$

$$V = \pi \times 4^3 \div 3 \times 8^3$$

$$= 2144.7\text{cm}^3$$

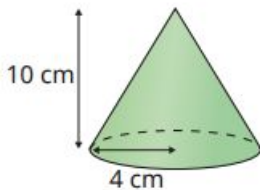


$$V = \frac{1}{3} \pi r^2 h$$

$$h = 10\text{cm} \quad r = 4\text{cm}$$

$$V = \pi \times 4^2 \times 10 \div 3$$

$$= 167.6\text{cm}^3$$

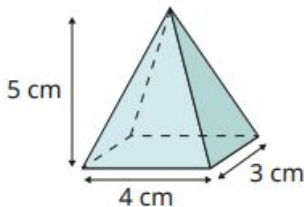


$$V = \frac{1}{3} h \times \text{area}$$

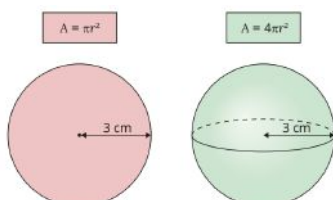
$$h = 5\text{cm}$$

$$\text{area} = 4 \times 3 = 12\text{cm}^2$$

$$V = 5 \times 12 \div 3 = 20\text{cm}^3$$



Surface area is the sum of the area of the visible surfaces on a shape. For most shapes, break them down into simple shapes, find their areas, and add them up.



Step 1

Isometric drawings

Step 2

Plans and elevations

Step 3

Volume of a cylinder

Step 4

Volume of a sphere

Step 5

Volume of cones and pyramids

Step 6

Surface area of a sphere

Step 7

Surface area of a cylinder (E)

Step 8

Surface area of a pyramid (E)

Step 9

Surface area of a cone (E)

Step 10

Convert metric units of area (E)

Step 11

Convert metric units of volume (E)

Key vocabulary

face	flat surface on a 3-D shape
edge	where two faces of a shape meet
vertex	point at which two or more edges (3-D) or sides (2-D) meet
plan	2-D diagram that shows the view of a 3-D shape from above
front elevation	2-D diagram that shows the view of a 3-D shape from the front
side elevation	2-D diagram that shows the view of a 3-D shape from the side
dimensions	measurements, such as length, width and height, that describe the size and extent of an object
surface area	area that covers the outside of a 3-D shape, calculated by finding the total area of all its surfaces
volume	amount of space a 3-D object takes up, measured in cubic units
radius	distance from the centre of a circle to any point of the circumference
diameter	straight line passing through the centre of the circle from one point on the circumference to another
metric	system of measurement based on units like metres, grams and litres, used to quantify length, mass and volume
unit	standard measurement used to express quantities

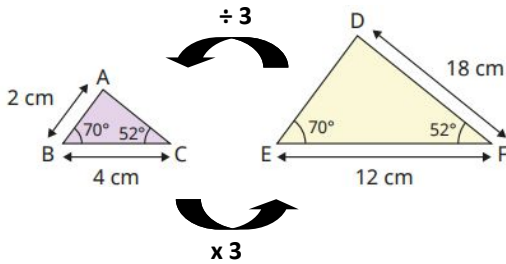


Autumn Block 3

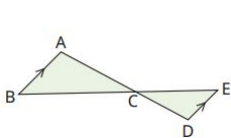
Similarity and congruence

Foundation Tier

Similar shapes are an enlargement of one another. We can use the scale factor to work out missing sides.



Similar triangles always have three pairs of equal angles. We can use a range of rules to show that pairs of sides are equal.



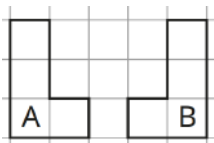
$\angle ACB = \angle ECD$ (vertically opposite angles are equal)

$\angle BAC = \angle CDE$ (alternate angles are equal)

ABC and CDE are **similar**.

$\angle ABC = \angle CED$ (alternate angles are equal)

Congruent shapes have the same side lengths and angles. They can only differ in orientation or position.



A and B are **congruent**.

We can prove that **triangles** are **congruent** if they have three matching elements using the **congruency criteria**.

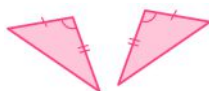
Side Side Side (**SSS**)



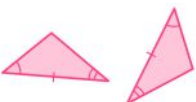
Angle Side Angle (**ASA**)



Side Angle Side (**SAS**)



Right-angle Hypotenuse



Side (**RHS**)

Step 1

Identify similar shapes

Step 2

Find unknown lengths and angles in similar shapes

Step 3

Identify similar triangles

Step 4

Solve problems with similar shapes

Step 5

Prove a pair of triangles are similar (E)

Step 6

Similarity and congruence

Step 7

Congruent triangles

Step 8

Prove a pair of triangles are congruent (E)

Key vocabulary

scale factor

number used to multiply the dimensions of a shape or object to make it larger or smaller compared to its original size

proportion

relationship between two or more quantities where the ratio of one quantity to another is the same

similar shapes

shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

corresponding

elements, such as angles or sides, that are in the same relative position

alternate angles

pairs of angles that are formed by parallel lines cut by a transversal, positioned on opposite sides of the transversal

corresponding angles

pairs of angles that are in the same relative position when parallel lines are cut by a transversal

congruent

identical to, in the context of comparing shapes or solids

proof

mathematical reasoning presented in logical steps to establish the truth of a statement beyond doubt

condition (congruence)

specific set of criteria that must be met for two shapes to be congruent































Autumn Block 1

Ratio and proportion

Higher Tier

When working with ratios, look carefully for which value(s) you've been given. Label each number carefully, so you don't get mixed up.

A

Mo and Ron share money in the ratio 3 : 5
Ron receives £50
How much does Mo receive?

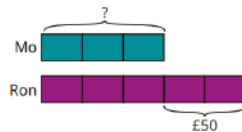


$$\begin{array}{c} M : R \\ 3 : 5 \\ \times 10 \quad \times 10 \\ \hline 30 : 50 \end{array}$$

Mo receives £30.

B

Mo and Ron share money in the ratio 3 : 5
Ron receives £50 more than Mo.
How much does Mo receive?



$$\begin{array}{c} 2 \text{ parts} = 50 \\ 1 \text{ part} = 25 \end{array} \div 2$$

$$\begin{array}{c} +2 \\ M : R \\ 3 : 5 \\ \times 25 \quad \times 25 \\ \hline 75 : 125 \end{array}$$

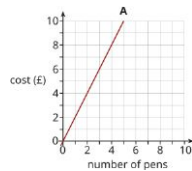
Mo receives £75.

In **direct proportion** relationships, both quantities are multiplied or divided by the same value. Examples:

Marking more books takes more time.

Marking less books takes less time.

OR

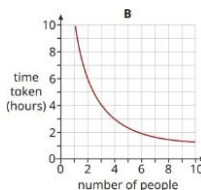


In **inverse proportion** relationships, we do the inverse operation to each value e.g. multiply one and divide the other.

Less cleaners take more time to clean a house.

More cleaners take less time to clean a house.

OR



We can also form equations for **direct** and **inverse** proportion relationships.

y is directly proportional to x.

When y = 100, x = 5

$$y = kx$$

$$100 = 5k$$

$$20 = k$$

$$y = 20x$$

y is directly proportional to x^2

When y = 100, x = 5

$$y = kx^2$$

$$100 = 25k$$

$$4 = k$$

$$y = 4x^2$$

We could use these equations to find the value of one variable, when given the other.

Step 1

Solve problems with ratio

Step 2

Best buy problems

Step 3

Conversion graphs

Step 4

Exchange rates

Step 5

Direct proportion equations

Step 6

Complex direct proportion equations (E)

Step 7

Inverse proportion

Step 8

Inverse proportion equations (E)

Step 9

Direct and indirect proportion graphs

Key vocabulary

ratio

comparison of two or more values

share

divide a quantity into equal-sized groups or parts

directly proportional

relationship between two quantities where they increase or decrease at the same rate

exchange rate

value at which one currency can be exchanged for another

currency

system of money used in a particular country

convert

change a quantity from one unit or form to another

inversely proportional

relationship between two quantities where, as one quantity increases at a rate, the other quantity decreases at the same rate

assumption

something accepted as true without proof

equation

statement to show that two expressions are equal

formula

mathematical rule or equation that shows the relationship between different variables or quantities

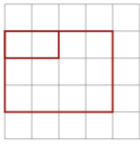
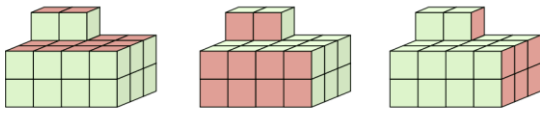
constant of proportionality

value that remains the same when two variables are directly or inversely proportional to each other



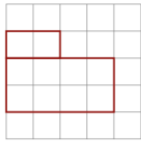
Autumn Block 2

Area and volume Higher Tier



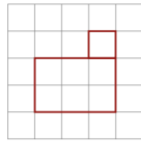
Plan

(view from above)



Front elevation

(view from the front)



Side elevation

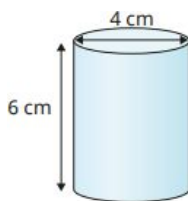
(view from the side)

$$V = \pi r^2 h$$

$$h = 6\text{ cm}$$

$$D = 4\text{ cm so } r = 2\text{ cm}$$

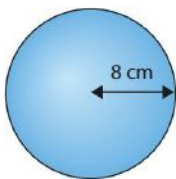
$$V = \pi \times 2^2 \times 6 = 75.4\text{ cm}^3$$



$$V = \frac{4}{3} \pi r^3$$

$$r = 8\text{ cm}$$

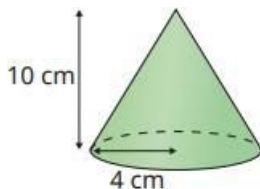
$$V = \pi \times 4^3 \div 3 = 214.7\text{ cm}^3$$



$$V = \frac{1}{3} \pi r^2 h$$

$$h = 10\text{ cm} \quad r = 4\text{ cm}$$

$$V = \pi \times 4^2 \times 10 \div 3 = 167.6\text{ cm}^3$$

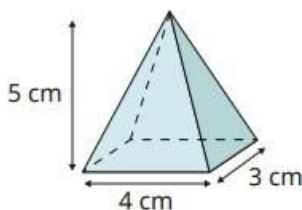


$$V = \frac{1}{3} h \times \text{area}$$

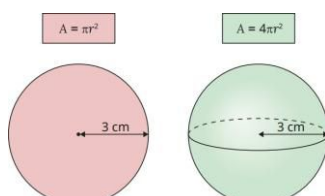
$$h = 5\text{ cm}$$

$$\text{area} = 4 \times 3 = 12\text{ cm}^2$$

$$V = 5 \times 12 \div 3 = 20\text{ cm}^3$$



Surface area is the sum of the area of the visible surfaces on a shape. For most shapes, break them down into simple shapes, find their areas, and add them up.



Step 1

Plans and elevations

Step 2

Volume of a sphere

Step 3

Volume of cones and pyramids

Step 4

Surface area of spheres and cones

Step 5

Volume and surface area of a frustrum (E)

Step 6

Convert metric units of area and volume

Key vocabulary

face	flat surface on a 3-D shape
edge	where two faces of a shape meet
vertex	point at which two or more edges (3-D) or sides (2-D) meet
plan	2-D diagram that shows the view of a 3-D shape from above
front elevation	2-D diagram that shows the view of a 3-D shape from the front
side elevation	2-D diagram that shows the view of a 3-D shape from the side
dimensions	measurements, such as length, width and height, that describe the size and extent of an object
surface area	area that covers the outside of a 3-D shape, calculated by finding the total area of all its surfaces
volume	amount of space a 3-D object takes up, measured in cubic units
radius	distance from the centre of a circle to any point of the circumference
diameter	straight line passing through the centre of the circle from one point on the circumference to another
metric	system of measurement based on units like metres, grams and litres, used to quantify length, mass and volume
unit	standard measurement used to express quantities
frustum	portion of a cone or pyramid remaining after its upper portion is removed following a cut off parallel to its base
similar shapes	shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other
perpendicular height	measure of how tall a shape is, measured at a right angle from the base to a vertex
slant height	non-perpendicular height between the base and the top of the solid



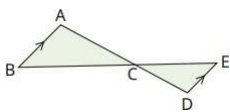
Autumn Block 3

Similarity and congruence

Higher Tier

Similar shapes are an enlargement of one another. We can use the scale factor to work out missing sides.

Similar triangles always have three pairs of equal angles. We can use a range of rules to show that pairs of sides are equal.



$\angle ACB = \angle ECD$ (vertically opposite angles are equal)

$\angle BAC = \angle CDE$ (alternate angles are equal)

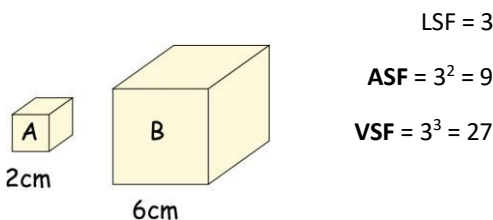
ABC and CDE are **similar**.

$\angle ABC = \angle CED$ (alternate angles are equal)

Similar shapes have different scale factors for length, area, and volume.

Area scale factor = Linear scale factor²

Volume scale factor = Linear scale factor³



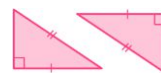
Congruent shapes have the same side lengths and angles. They can only differ in orientation or position.

We can prove that **triangles** are **congruent** if they have three matching elements using the **congruency criteria**.

Side Side Side (SSS)



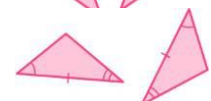
Angle Side Angle (ASA)



Side Angle Side (SAS)



Right-angle Hypotenuse



Side (RHS)

Step 1

Find unknown lengths and angles in similar shapes

Step 2

Identify similar triangles

Step 3

Solve problems with similar shapes

Step 4

Prove a pair of triangles are similar

Step 5

Areas of similar shapes

Step 6

Volumes of similar shapes

Step 7

Solve problems with area and volume in similar shapes

Step 8

Congruence

Step 9

Prove a pair of triangles are congruent

Key vocabulary

scale factor

number used to multiply the dimensions of a shape or object to make it larger or smaller compared to its original size

proportion

relationship between two or more quantities where the ratio of one quantity to another is the same

similar shapes

shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

corresponding

elements, such as angles or sides, that are in the same relative position

alternate angles

pairs of angles that are formed by parallel lines cut by a transversal, positioned on opposite sides of the transversal

corresponding angles

pairs of angles that are in the same relative position when parallel lines are cut by a transversal

congruent

identical to, in the context of comparing shapes or solids

proof

mathematical reasoning presented in logical steps to establish the truth of a

condition (congruence)

statement beyond doubt
specific set of criteria that must be met for two shapes to be congruent



Autumn Block 4

Sequences and proof

Higher Tier

We can find the **nth term** of a linear sequence by observing the common difference and the offset.

The sequence below has a common difference of **+2** and an offset of **+1**. So, it has an nth term of **$2n + 1$** .

$$\begin{array}{ccccccc} & & -2 & & & & \\ & & \curvearrowright & & & & \\ \text{B: } & 1 & 3, & 5, & 7, & 9, & 11 \dots \\ & & +2 & +2 & +2 & +2 & \end{array}$$

We can find the **nth term** of a quadratic sequence by working with second differences.

$$\begin{array}{ccccccc} 4, & 15, & 32, & 55, & \dots \\ +11 & +17 & +23 & & \\ +6 & +6 & & & \end{array}$$

$6 \div 2 = 3$, so our nth term involves **$3n^2$** .

$$\begin{array}{r} \text{Original sequence: } 4, 15, 32, 55, \dots \\ \text{ } 3n^2: \quad \quad \quad -1 \quad 3, 12, 27, 48, \dots \\ \hline \quad \quad \quad 1, 3, 5, 7, \dots \\ \quad \quad \quad +2 \quad +2 \quad +2 \end{array}$$

When we subtracted $3n^2$ from the original sequence, we were left with **$2n - 1$** . So, our quadratic nth term is **$3n^2 + 2n - 1$** .

Sequences can involve **surds**.

Linear sequence:

$$\begin{array}{ccccccc} 3\sqrt{2} + 4, & 5\sqrt{2} + 4, & 7\sqrt{2} + 4, & \dots \\ +2\sqrt{2} & +2\sqrt{2} & & \end{array}$$

Geometric sequence:

$$\begin{array}{ccccccc} 3\sqrt{5}, & 15, & 15\sqrt{5}, & \dots \\ \times \sqrt{5} & \times \sqrt{5} & & \end{array}$$

To construct **algebraic proofs**, we need to represent numbers with algebra. Here are some ideas you can use:

$$\begin{array}{l} n \rightarrow \text{an integer} \\ 2n \rightarrow \text{an even integer} \\ 2n + 1 \rightarrow \text{an odd integer} \\ 3(\text{integer}) \rightarrow \text{a multiple of 3} \end{array}$$

Step 1

Describe and continue sequences

Step 2

Explore sequences with algebraic terms

Step 3

Describe and continue sequences involving surds (E)

Step 4

Generate a sequence given an algebraic rule

Step 5

nth term of a linear sequence

Step 6

nth term of a quadratic sequence (E)

Step 7

Represent numbers algebraically

Step 8

Algebraic arguments and proof

Key vocabulary

arithmetic sequence	sequence with a common difference between consecutive terms
geometric sequence	sequence with a common ratio between consecutive terms
Fibonacci sequence	sequence where the next term is found by adding the previous two terms together
triangular number	number that is the sum of all positive integers up to a given integer, for example, $10 = 1 + 2 + 3 + 4$
surd	irrational number expressed as the root of a positive integer
simplify	extracting square factors of the number within the root
term	number/object that relates to a specific position in a sequence
term-to-term rule	rule that describes how you get from one term of a sequence to the next
nth term rule	algebraic rule that generalises all terms in the sequence
quadratic	equation or expression involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$
second difference	difference between the first differences of a sequence
consecutive	one after another in order
expression	collection of terms involving variables and numbers
identity	equation that is true for all values of the variable, for example, $a + b \equiv b + a$

























