



Autumn Block 1

Algebraic manipulation Higher Tier

definition	characteristics
terms with the same variable(s) and power(s)	- Same variable(s) - Same power
examples	non-examples
x $3x$ $\frac{x}{7}$ $\frac{1}{2}x$ $-5x$	$6xy$ $4x^2$ 8^x $\frac{3}{x}$

Like terms

$$4^5 \times 4^3 = 4^8$$

$$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^8$$

$$\frac{4^5}{4^3} = 4^2$$

$$\frac{\cancel{4} \times \cancel{4} \times \cancel{4} \times 4 \times 4}{\cancel{4} \times \cancel{4} \times \cancel{4}} = 4^2$$

$$(4^5)^3 = 4^{15}$$

$$\begin{aligned}
 4^5 \times 4^5 \times 4^5 &= 4^{5+5+5} \\
 &= 4^{3 \times 5} \\
 &= 4^{15}
 \end{aligned}$$

Expand the brackets.

$$2(m + 5) \equiv 2m + 10$$

$$2 \times m = 2m$$

$$2 \times 5 = 10$$

Factorise the expressions fully.

$$12n^2 + 15n \equiv 3n(4n + 5)$$

$$\text{HCF} = 3n$$

$$12n^2 \div 3n = 4n$$

$$15n \div 3n = 5$$

Step 1

Simplify expressions

Step 2

Laws of indices

Step 3

Expand a single bracket

Step 4

Factorise into a single bracket

Key vocabulary

expression collection of terms involving variables (letters) and numbers

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

formula mathematical rule or equation that shows the relationship between different variables or quantities

like terms terms with the same variable(s) and power(s)

coefficient number in front of a variable indicating the multiple of the variable

simplify write an expression in a simpler equivalent form

power way to express repeated multiplication of a number using a base and an index

base number that is raised to a certain index, for example, 10^3 has a base of 10

index written as a small number to the right and above the base number, which indicates how many times the base has been multiplied by itself

indices plural of index

expand multiply each term inside the bracket by the expression outside the bracket

highest common factor (HCF) common factor of two or more integers or terms that has the greatest value or power



Autumn Block 2

Equations, inequalities and formulae Higher Tier

Solve $3m - 9 = 12$

$$3m - 9 = 12$$

$$+ 9 \quad + 9$$

$$3m = 21$$

$$\div 3 \quad \div 3$$

$$m = 7$$

Solve $5t + 3 = 3t + 17$

$$5t + 3 = 3t + 17$$

$$-3t \quad -3t$$

$$2t + 3 = 17$$

$$-3 \quad -3$$

$$2t = 14$$

$$\div 2 \quad \div 2$$

$$t = 7$$

Make b the subject of the formula.

$$A = \frac{1}{2}bh$$

$$A = \frac{bh}{2}$$

$$\times 2 \quad \times 2$$

$$2A = bh$$

$$\div h \quad \div h$$

$$\frac{2A}{h} = b$$

Step 1

Solve equations

Step 2

Solve fractional equations

Step 3

Solve equations with unknowns on both sides

Step 4

Understand inequalities

Step 5

Solve inequalities

Step 6

Represent solutions to inequalities using set notation (E)

Step 7

Change the subject of a simple formula

Step 8

Change the subject of a known formula

Step 9

Change the subject of a complex formula

Step 10

Change the subject where the subject appears more than once (E)

Key vocabulary

equation statement to show that two expressions are equal

inverse opposite effect of

expression collection of terms involving variables and numbers

unknown value not yet known, indicated with a letter and used in an equation or inequality

solution value of an unknown that satisfies an equation or inequality

coefficient number in front of a variable indicating the multiple of the variable

integer whole number that can be positive or negative

inequality mathematical statement that shows the relationship between two expressions where one is greater than, less than, or not equal to the other

satisfy find a value that makes the inequality true when substituted into the inequality

solve find the value(s) or set of values of an unknown in an equation or inequality

formula mathematical rule or equation that shows the relationship between different variables or quantities.

subject variable that is isolated on one side of the formula

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

rearrange adjust a formula to isolate a specific variable or simplify an expression

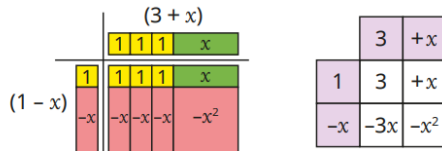


Autumn Block 3

Quadratic expressions and equations

Higher Tier

$$(1-x)(3+x) \equiv 3-2x-x^2$$



$$(x+2)(x+4)(x+3)$$

$$(x+2)(x+4)(x+3) \equiv (x^2+6x+8)(x+3)$$

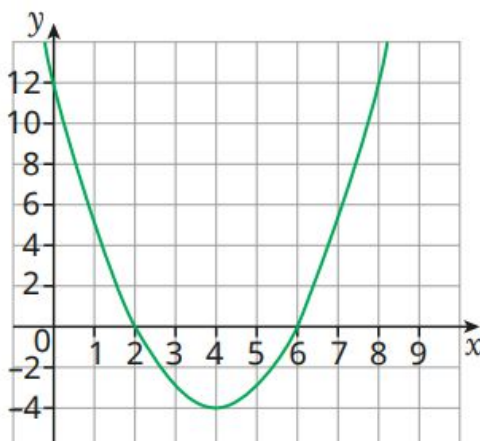
	x	3
x^2	x^3	$3x^2$
$6x$	$6x^2$	$18x$
8	$8x$	24

$$x^3+3x^2+6x^2+18x+8x+24 \equiv x^3+9x^2+26x+24$$

$$\text{Solve } x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$x-2=0 \quad \text{or} \quad x-6=0$$



Step 1

Expand double brackets

Step 2

Expand triple brackets

Step 3

Factorise quadratic expressions

Step 4

Factorise more complex quadratic expressions (E)

Step 5

Difference of two squares

Step 6

Solve quadratic equations equal to 0

Step 7

Solve quadratic equations by factorisation

Step 8

Solve more complex quadratic equations by factorisation (E)

Step 9

Complete the square

Step 10

Complete the square with more complex quadratic expressions (E)

Step 11

Solve quadratic equations by completing the square (E)

Step 12

Solve quadratic equations using the quadratic formula

Key vocabulary

quadratic

equation or expression involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$

expand

multiply each term inside the bracket by the expression outside the bracket

term

number or variable or any combination of number and variable combined by multiplication or division

sum

result of adding two or more numbers together

product

result of a multiplication

factorise

express an algebraic expression as a product of its factors using brackets; the reverse process of expanding brackets

variable

symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

expression

collection of terms involving variables and numbers

solve

find the value(s) or set of values of an unknown in an equation or inequality

solution

value of an unknown that satisfies an equation or inequality

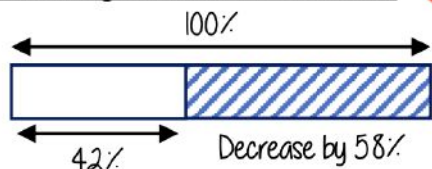


Autumn Block 4

Percentages

Higher Tier

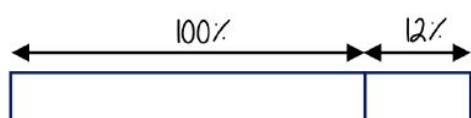
Percentage increase/decrease



$$100\% - 58\% = 42\%$$

$$100 - 58 = 0.42$$

Multiplier
Less than 1



$$100\% + 12\% = 112\%$$

$$100 + 0.12 = 1.12$$

Multiplier
More than 1

Simple Interest

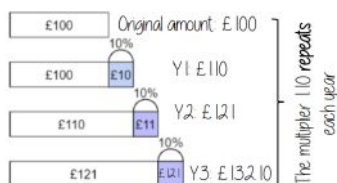
James invests
£2000 at 5%
simple interest



The original value increases
by this amount every year

Compound Interest

Tess invests
£100 at 10%
compound
interest for 3
years



Find the original value

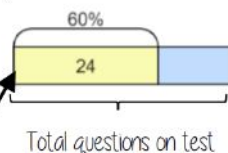
In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test?

$$\text{Original} \times 0.6 = 24$$

$$24 \div 0.6 = 40 \text{ marks}$$

$$10\% = 6$$

$$100\% = 40$$



Step 1

Percentage of an amount

Step 2

Percentage increase and decrease

Step 3

Repeated percentage change

Step 4

Express one number as a fraction or percentage of another

Step 5

Express a change as a percentage

Step 6

Find the original value after a percentage change

Step 7

Simple interest

Step 8

Compound interest

Step 9

Choose appropriate methods to solve percentage problems

Key vocabulary

percentage expressing a part of a whole as a number out of 100

decimal number number with a whole and fractional part, for example 3.2, 5.43

multiplier number that is multiplied with another number to increase or decrease its value

depreciate decrease in value over time

fraction number with a numerator and a denominator that represents a proportion

equivalent fraction fraction that represents the same value as another fraction with different numerators and denominators

percentage change how much a quantity has increased or decreased relative to its original value, calculated as a percentage

profit positive difference between the selling price and the cost price of an item, indicating financial gain

loss negative difference between the cost price and the selling price of an item, indicating a financial loss

simple interest interest calculated on the principal amount and then multiplied by the time period of the investment or loan

deposit money placed in a bank account or given as an initial payment for a service or purchase

investment amount deposited in a savings account with the goal of earning a financial gain or increasing the value over time

per annum "per year", used to describe interest rates or payments that occur annually

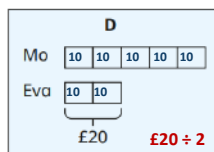
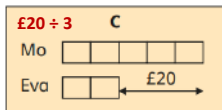
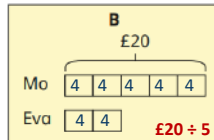
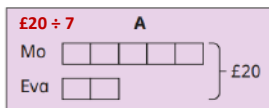
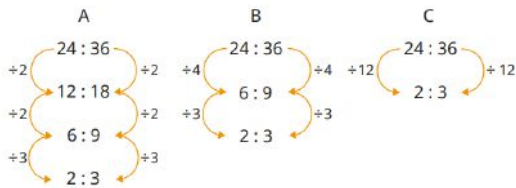
compound interest interest calculated on both the principal amount and the accumulated interest from previous periods



Autumn Block 5

Ratio and scale

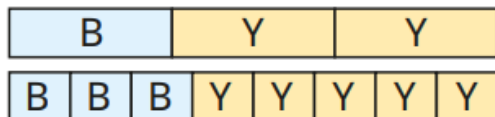
Higher Tier



Max and Dexter make green paint by mixing blue and yellow paint together.

Max
blue : yellow = 1 : 2

Dexter
blue : yellow = 3 : 5



Who makes a darker shade of green paint?

Red	Blue	Yellow
R R	B B B	
	B B B	Y

R : B : Y
2 : 3
6 : 1

Red	Blue	Yellow
R R	B B B	
R R R	B B B B	Y

R : B : Y
 $\times 2$ 2 : 3 $\times 2$
4 : 6 : 1

The ratio of x to y is 2 : 3
 $x + y = 30$
Work out the values of x and y.

x

y

30

$30 \div 5 = 6$

$x = 2 \times 6$ $y = 3 \times 6$
 $x = 12$ $y = 18$

Step 1

Equivalent ratios

Step 2

Share in a ratio (whole given)

Step 3

Share in a ratio (part or difference given)

Step 4

Link ratios and fractions

Step 5

Combine a set of ratios (E)

Step 6

Share in a ratio (algebraically) (E)

Step 7

Ratios and scales

Key vocabulary

highest common factor common factor of two or more integers or terms that has the greatest value or power

simplest form when the highest common factor of the values in a ratio is 1

equivalent ratio two ratios are equivalent if they simplify to the same ratio

ratio comparison of two or more values

fraction number with a numerator and a denominator that represents a proportion

numerator top number in a fraction that indicates the number of equal parts out of a whole

denominator bottom number in a fraction that indicates the number of equal parts a whole has been divided into

lowest common multiple lowest multiple shared by two or more numbers

scaling multiplication of each value by a constant to proportionally increase or decrease the size

common part same part represented in two ratios, for example, in the ratios red : blue and blue : green, the common part is blue

algebraic related to algebra, often involving expressions, equations, or symbols representing numbers or quantities

expression collection of terms involving variables (letters) and numbers



Autumn Block 6

Work with fractions

Higher Tier

Method A

$$5\frac{1}{3} \div 4\frac{4}{9} = \frac{16}{3} \div \frac{40}{9} = \frac{16}{3} \times \frac{9}{40} \\ = \frac{144}{120} = 1\frac{24}{120} = 1\frac{1}{5}$$

Method B

$$5\frac{1}{3} \div 4\frac{4}{9} = \frac{16}{3} \div \frac{40}{9} = \frac{16^2}{3^1} \times \frac{9^3}{40^5} \\ = \frac{2}{1} \times \frac{3}{5} = \frac{6}{5} = 1\frac{1}{5}$$

$$\frac{2}{7} \times 3 = \frac{6}{7} \quad \text{and} \quad \frac{a}{7} \times 3 \equiv \frac{3a}{7}$$

$$3 \times \frac{5}{8} = \frac{15}{8} \quad \text{and} \quad 3 \times \frac{5}{a} \equiv \frac{15}{a}$$

$$\frac{2}{3} \times 4 = \frac{8}{3} \quad \text{and} \quad \frac{2}{3} \times a \equiv \frac{2a}{3}$$

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15} \quad \text{and} \quad \frac{a}{b} \times \frac{c}{d} \equiv \frac{a \times c}{b \times d} \equiv \frac{ac}{bd}$$

$$\frac{2}{7} \div 3 = \frac{2}{21} \quad \text{and} \quad \frac{a}{7} \div 3 \equiv \frac{a}{21}$$

$$\frac{5}{8} \div 3 = \frac{5}{24} \quad \text{and} \quad \frac{5}{a} \div 3 \equiv \frac{5}{3a}$$

$$\frac{2}{5} \div 3 = \frac{2}{15} \quad \text{and} \quad \frac{2}{5} \div a \equiv \frac{2}{5a}$$

$$\frac{x^2 - 4}{x + 2} \times \frac{x + 2}{x - 2} \equiv \frac{(x + 2)(x - 2)}{\cancel{x + 2}} \times \frac{\cancel{x + 2}}{\cancel{x - 2}} \equiv x + 2$$

Key vocabulary

product	result of a multiplication
reciprocal	one of a pair of numbers whose product is equal to 1
factor	integer or term that divides exactly into another integer or term
factorise	express an algebraic expression as a product of its factors using brackets; the reverse process of expanding brackets
common denominator	when two or more fractions have the same denominator

Step 1

Add and subtract fractions

Step 2

Multiply and divide fractions

Step 3

Solve problems with fractions

Step 4

Add and subtract algebraic fractions

Step 5

Multiply algebraic fractions

Step 6

Divide algebraic fractions

Step 7

Simplify algebraic fractions by factorising

Step 8

Add and subtract more complex algebraic fractions (E)

Step 9

Multiply and divide more complex algebraic fractions (E)

Step 10

Solve equations with algebraic fractions (E)

Key vocabulary

mixed number	integer and a proper fraction represented together to form a number
improper fraction	fraction where the numerator is greater than the denominator
equivalent fraction	fraction that represents the same value as another fraction with different numerators and denominators
reciprocal	one of a pair of numbers whose product is equal to 1
numerator	top number in a fraction that indicates the number of equal parts out of a whole
denominator	bottom number in a fraction that indicates the number of equal parts a whole has been divided into
algebraic fraction	fraction where the numerator, the denominator, or both are algebraic expressions
lowest common multiple (LCM)	lowest multiple shared by two or more numbers
simplify	write an expression in a simpler equivalent form

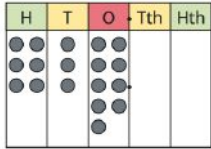


Spring Block 1

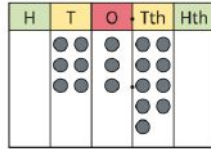
Non-calculator methods

Higher Tier

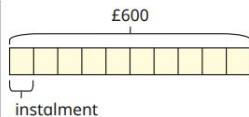
$$3 \times 213 = 639$$



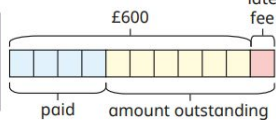
$$3 \times 21.3 = 63.9$$



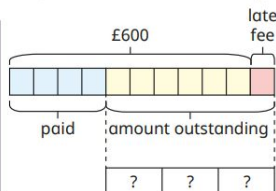
Mr Fisher buys a laptop for £600
He plans to pay in 10 equal instalments.



He pays the 4th instalment late and gets a £15 late fee.



He wants to pay the remaining amount plus the late fee in 3 equal instalments.
How much money must he pay per instalment?



$$x = 0.37777...$$

$$10x = 3.77777...$$

$$100x = 37.77777...$$

$$\begin{array}{r} 100x = 37.77777... \\ - 10x = 3.77777... \\ \hline 90x = 34 \\ x = \frac{34}{90} = \frac{17}{45} \end{array}$$

$$x = 0.1575757...$$

$$10x = 1.5757575...$$

$$100x = 15.7575757...$$

$$1000x = 157.5757575...$$

$$\begin{array}{r} 1000x = 157.5757575... \\ - 10x = 1.5757575... \\ \hline 990x = 156 \\ x = \frac{156}{990} = \frac{26}{165} \end{array}$$

Step 1

Order of operations

Step 2

Related calculations

Step 3

Solve multi-step problems

Step 4

Convert recurring decimals to fractions

Step 5

Convert more complex recurring decimals to fractions (E)

Key vocabulary

commutative when an operation gives the same result whatever the order of the terms involved

inverse opposite effect of

deposit money placed in a bank account or given as an initial payment for a service or purchase

instalments regular, smaller payments made over time to pay back a greater amount of money

profit positive difference between the selling price and the cost price of an item, indicating financial gain

interest amount of money charged or earned based on a percentage of the original amount over a period of time

recurring decimal decimal number that has a single or group of digits infinitely repeating, usually written with a dot above the repeating digit or start and end of the group of repeating digits

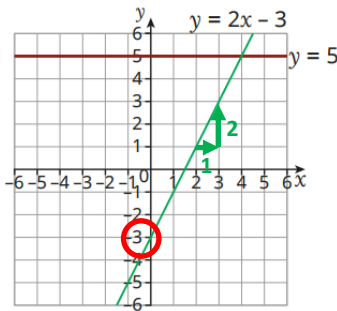
terminating decimal decimal number that has a finite number of digits



Spring Block 2

Straight line graphs

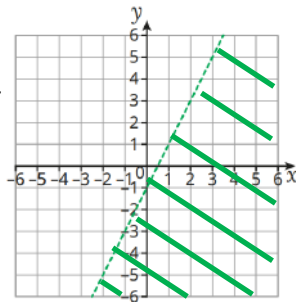
Higher Tier



$y = 2x - 3$
gradient = 2
y-intercept = -3

Shade the region satisfied by the inequality:

$$y < 2x - 1$$



Line B has a gradient of 4 and goes through the point (3, 7).

$$y = mx + c$$

The gradient is 4, so $m = 4$

Substitute into $y = mx + c$.

$$y = 4x + c$$

The line goes through the point (3, 7).

Substitute and solve.

$$7 = 4(3) + c$$

$$7 = 12 + c$$

$$-5 = c$$

Hence, the equation of the line is $y = 4x - 5$

(3, 7)

(5, 11)

$$\frac{11 - 7}{5 - 3}$$

$$y = 2x + c$$

(3, 7)
x y

$$7 = 2 \times 3 + c$$

$$7 = 6 + c$$

$$1 = c$$

$$\rightarrow y = 2x + 1$$

Step 1

Draw straight line graphs

Step 2

$$y = mx + c$$

Step 3

Find the equation of a line from a graph

Step 4

Represent solutions to single inequalities in a graph

Step 5

Represent solutions to multiple inequalities on a graph (E)

Step 6

Find the midpoint of a line segment

Step 7

Equation of a straight-line graph given one point and a gradient

Step 8

Equation of a straight-line graph given two points

Step 9

Equations of perpendicular lines (E)

Step 10

Real-life straight-line graphs

Key vocabulary

linear

where a relationship or pattern changes at a constant rate, forming a straight line when graphed

gradient

measure of how steep a line is, showing the rate of change between two points on a graph

y-intercept

point where a line or curve crosses the y-axis on a graph

parallel lines

lines that maintain a constant distance from each other

inequality

mathematical statement that shows the relationship between two expressions where one is greater than, less than, or not equal to the other

midpoint

halfway between two intervals on a number line or two numbers

line segment

part of a line between two points

mean

type of average found by dividing the total of the values by the number of values in the set

perpendicular

lines, planes or surfaces that meet at a right angle

negative reciprocal

one of a pair of numbers whose product is equal to -1



Spring Block 3

Probability

Higher Tier

Colour	red	blue	green
Probability	0.3	0.2	

Probabilities sum to 1, so:

$$P(\text{green}) = 1 - (0.3 + 0.2) = 0.5$$

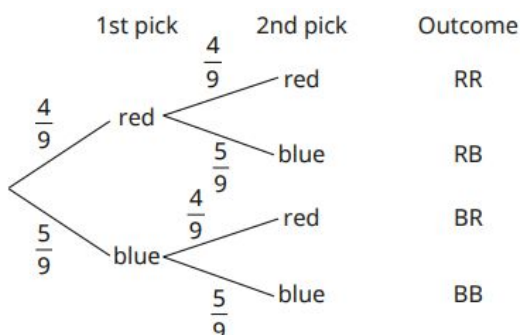
Relative frequency =

$\frac{\text{number of successes}}{\text{number of trials}}$

A		B						
(T, 1) (T, 2)		Dice						
(T, 3) (T, 4)		1	2	3	4	5	6	
(T, 5) (T, 6)								
(H, 1) (H, 2)	Coin	Heads	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
(H, 3) (H, 4)		Tails	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)
(H, 5) (H, 6)								

A and B are examples of sample space diagrams for rolling a dice and flipping a coin.

A bag contains 4 red and 5 blue counters. A counter is selected at random, its colour is noted and then it is replaced. Then another counter is selected.



Step 1

Probability of a single event

Step 2

Use the property that probabilities sum to 1

Step 3

List outcomes

Step 4

Relative frequency

Step 5

Sample spaces for 1 or more events

Step 6

Two-way tables

Step 7

Frequency trees

Step 8

Independent events

Step 9

Tree diagrams for independent events

Step 10

Tree diagrams for dependent events (E)

Key vocabulary

probability measure of how likely an event is to happen

event outcome or group of outcomes that have a probability of happening

outcome possible result of an experiment or trial

experimental probability likelihood of an event based on results from trials

theoretical probability expected likelihood of an event happening

frequency number of times an outcome appears in a dataset

expected frequency number of times an event is predicted to occur, based on its theoretical probability or relative frequency

relative frequency number of times an event occurs compared to the total number of trials

sample space complete set of all possible outcomes of a probability experiment

independent one event does not affect the probability of the other event occurring

dependent one event affects the probability of the other event occurring



Spring Block 4

Rounding and estimation

Higher Tier

Round 5.63705
to one decimal place.

5.6|3705 Keep 1 digit after the decimal point

5.6|[↓]3705 The next digit tells us to round down, because it's less than 5

5.6 We remove the rest of the digits, as we only want 1 decimal place

Round 36,271 to three significant figures.

36,2|71 Keep 3 digits from the first significant figure (non-zero digit)

36,3|[↑]71 The next digit tells us to round up, because it's 5 or greater

36,300 We replace the rest of the digits with zeros, as they affect the value of the number

Work out an estimate for the value of 54×9.2

A	B	C	D
54×9	50×10	50×9	55×10

An estimate finds an approximate answer to a question. We should round to 1 significant figure to find an estimate in most cases.

Error intervals

$x = 3.4$ rounded to 1 decimal place	$3.4 \leq x < 3.5$
$x = 3.44$ rounded to 3 significant figures	$3.35 \leq x < 3.45$
$x = 3.4$ truncated to 1 decimal place	$3.435 \leq x < 3.445$

Step 1

Round to decimal places and significant figures

Step 2

Estimate answers to calculations

Step 3

Use of a calculator

Step 4

Error intervals (including truncation)

Step 5

Upper and lower bounds

Key vocabulary

round replacing a number with an approximate value

significant figures digits in a number starting from the first non-zero digit

decimal place position of a digit to the right of the decimal point

degree of accuracy indication of how precise a number is

estimate an appropriate value

approximate as close to the actual value as possible but not completely accurate

overestimate an estimate that is greater than the actual value

underestimate an estimate that is less than the actual value

truncate shorten a number by removing digits beyond a certain point without rounding

limits minimum and maximum values, often stated with inequalities to indicate whether the limit can be included

upper bound upper limit of the set of values that rounds to a number, which is stated to a given degree of accuracy

lower bound lower limit of the set of values that rounds to a number, which is stated to a given degree of accuracy

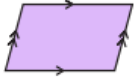
error interval range of values a number could be, to a degree of accuracy



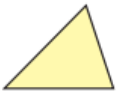
Spring Block 5

Perimeter, area and volume

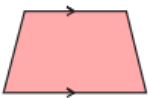
Higher Tier



base $\times$ perpendicular height



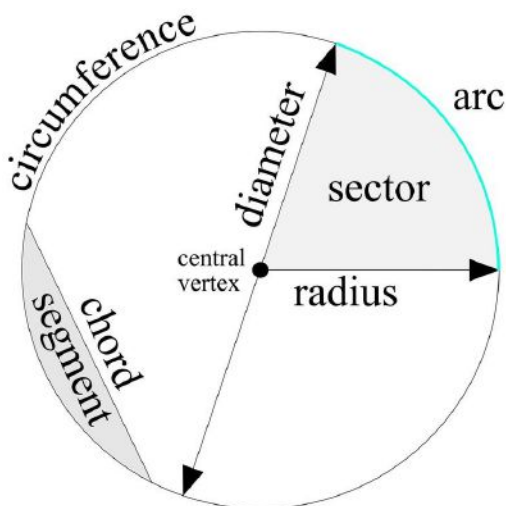
$\frac{1}{2} \times$ base $\times$ height



$\frac{1}{2} (a + b)h$



length $\times$ width



$$\text{Arc length} = \frac{\theta}{360} \times \pi \times D$$



$$\text{Sector area} = \frac{\theta}{360} \times \pi \times r^2$$



Step 1

Name 2-D and 3-D shapes

Step 2

Perimeter of a 2-D shape

Step 3

Area of a 2-D shape

Step 4

Area of a compound shape

Step 5

Recognise and label parts of a circle

Step 6

Circumference of a circle

Step 7

Area of a circle

Step 8

Volume of a prism

Step 9

Nets

Step 10

Surface area of a prism

Key vocabulary

radius

distance from the centre of a circle to any point of the circumference

diameter

straight line passing through the centre of a circle from one point on the circumference to another

arc

proportion of the circumference of a circle

sector

part of a circle formed from two radii subtended by an arc

chord

line segment that connects any two points on the circle's circumference

segment

region enclosed by a chord and its arc

perpendicular height

height of a shape, measured at a right angle from the base to a vertex or edge

face

flat surface on a 3-D shape

edge

straight line between two vertices or faces of a shape

vertex

point at which two or more edges (3-D) or sides (2-D) meet



Spring Block 6

Interpret and represent data

Higher Tier

Test score, s	x	f	fx
40 ≤ s < 50	45	8	360
50 ≤ s < 60	55	5	275
60 ≤ s < 70	65	11	715
70 ≤ s < 80	75	6	450
		30	1800

$$\text{Estimated Mean} = \frac{\sum fx}{\sum f} = \frac{1800}{30} = 60$$

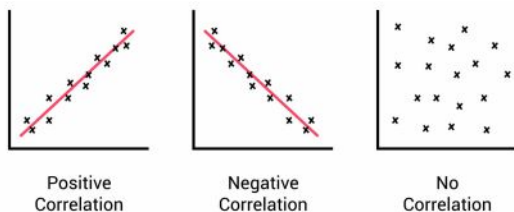
Modal class = $60 \leq s < 70$
(the interval with the highest frequency)

$$\text{Class containing median} = \frac{\sum f + 1}{2} \text{th value}$$

$$= \frac{30 + 1}{2} = 15.5 \text{th value} = 60 \leq s < 70$$

	Mean time (seconds)	Range of times (seconds)
Runner C	15.5	1.5
Runner D	14.1	1.5

Runner D is faster than runner C on average, because they had a lower mean finish time. Both runners are equally consistent, as they have the same range of finish times.



Key vocabulary

population	whole group that the data is collected from or analysed in a statistical investigation
sample	smaller group taken from a population to represent the whole in a statistical study
stratified sample	sample taken by dividing the population into groups and selecting a random sample from each group in proportion to its size
census	survey that collects data from every member of a population
random sample	group chosen from a population where every member has an equal chance of being selected
interpolation	estimating a value within the range of known data points
extrapolation	estimating a value beyond the range of known data points

Step 1	Averages and range
Step 2	Averages from an ungrouped frequency table
Step 3	Mean from a grouped frequency table
Step 4	Averages from a grouped frequency table
Step 5	Use data to compare distributions
Step 6	Types of data
Step 7	Sampling
Step 8	Capture and recapture
Step 9	Scatter graphs
Step 10	Interpolation and extrapolation

Key vocabulary

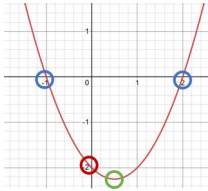
average	number or item representing the typical value of a set of data
mean	type of average found by dividing the total of the values by the number of values in the set
median	type of average found by identifying the middle number in an ordered list
mode	type of average found by identifying the item that has the greatest frequency in a data set
range	difference between the greatest and smallest pieces of data in a set
frequency	number of times a value appears in a data set
modal class	class interval with the greatest frequency
cumulative frequency	running total of frequencies, showing the number of data points less than or equal to each class interval
discrete data	data that can be counted
continuous data	measured data that can take any value, including decimals
qualitative data	non-numerical data
primary data	data collected by the researcher through methods like surveys, experiments or observations
secondary data	data collected by someone else, such as from websites, reports or books



Spring Block 7

Non-linear graphs

Higher Tier



Roots: $x = -1$ and $x = 2$

y-intercept: $(0, -2)$

Turning point: $(0.5, -2.25)$

$$y = x^2 - x - 2$$

Can you see the y-intercept in the equation, too?

$$(x - 2)(x + 1) = 0$$

What about the roots?

The **gradient of a line** shows how steep the straight line is. In the general equation of straight line, $y = mx + c$, the gradient is denoted by the letter m .

To calculate the gradient of a straight line through two coordinates (x_1, y_1) and (x_2, y_2) :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

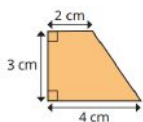
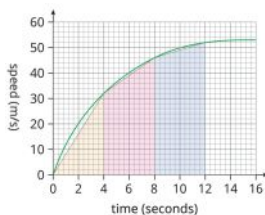
Example

$$m = \frac{5 - 1}{4 - 2} = \frac{4}{2} = 2$$

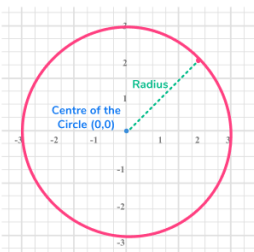


It can be helpful to think about this formula as: "change in y divided by change in x " or "rise over run".

We can estimate the area under a curve by splitting the area into equal width trapezia. Find the area of each trapezium, then add them together.

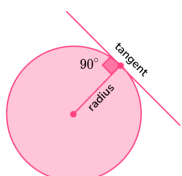


$$\text{area} = \underline{a+b} \times h$$



The equation of a circle with centre $(0, 0)$ is $x^2 + y^2 = r^2$, where r is the length of the radius.

The **equation of a tangent** is the equation of the straight line touching the circumference of a circle at only one point.



m_1 = gradient of radius
We find this using the gradient formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

m_2 = gradient of tangent
Tangent perpendicular to radius: $m_1 m_2 = -1$

$$m_2 = \frac{-1}{m_1}$$

negative reciprocal

Finally, find the y-intercept of the tangent for the full equation.

Step 1

Quadratic graphs

Step 2

Intercepts and roots of quadratic graphs

Step 3

Turning points

Step 4

Cubic graphs

Step 5

Approximate solutions to equations using graphs

Step 6

Estimate the gradient of a curve at a given point

Step 7

Estimate the area under a curve (E)

Step 8

Equation of a circle

Step 9

Equation of a tangent to a circle (E)

quadratic

involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$

intercept

point where a line or curve crosses an axis on a graph

root

value(s) of x where the quadratic equals 0

solution

value of an unknown that satisfies an equation or inequality

turning point

point where the gradient of a graph changes from positive to negative or vice versa

cubic

involving the third (and no higher) power of a variable or unknown, typically in the form $ax^3 + bx^2 + cx + d$

intersect

where straight lines or curves meet and cross at a point

tangent

straight line that touches a curve at exactly one point, but does not cross that point

gradient

measure of how steep a line is, showing the rate of change between two points on a graph

radius

distance from the centre of a circle to any point of the circumference

origin

where the axes of a graph intersect, at coordinates $(0, 0)$

negative reciprocal

one of a pair of numbers whose product is equal to -1



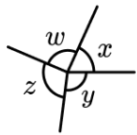
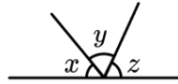
Summer Block 1

Angles

Higher Tier

The sum of angles on a straight line is 180° .

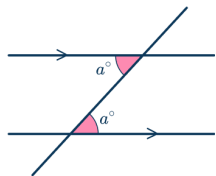
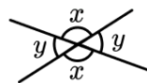
$$x + y + z = 180$$



The sum of angles at a point is 360° .

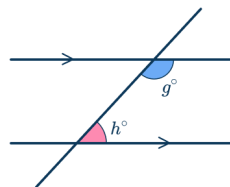
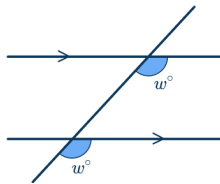
$$w + x + y + z = 360$$

Vertically opposite angles are equal in size.



Alternate angles are equal (z-shape).

Corresponding angles are equal (f-shape).



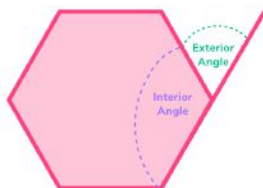
Co-interior angles add to 180° (c-shape).

Interior angles in a triangle add to 180° .



Interior angles in a quadrilateral add to 360° .

An adjacent interior and exterior angle add to 180° .



Exterior angles in any polygon add to 360° .

$$\text{Sum of interior angles} = (n - 2) \times 180^\circ$$

Step 1

Angles around a point, on a straight line and vertically opposite

Step 2

Angles in triangles and quadrilaterals

Step 3

Exterior angles of a polygon

Step 4

Interior angles of a polygon

Step 5

Solve problems with angles in polygons

Step 6

Co-interior, alternate and corresponding angles

Step 7

Solve problems with angles and algebra

Step 8

Prove geometric facts (E)

Key vocabulary

vertically opposite

pair of equal angles that are opposite each other at a vertex created by two straight intersecting lines

triangle

polygon with three straight sides and three interior angles that sum to 180°

quadrilateral

polygon with four straight sides and four angles that sum to 360°

isosceles triangle

triangle with two equal sides and angles

right angle

angle that measures 90° and can be marked by a square marker

exterior angle

angle formed when one side of a polygon is extended, creating an angle outside the polygon

interior angle

angle inside a polygon where two sides meet

polygon

closed 2-D shape made of straight line segments

regular polygon

polygon where all the sides are equal in length and all the interior angles are equal

parallel lines

lines that maintain a constant distance from each other

transversal

line that crosses two or more other lines at distinct points, often creating pairs of angles

co-interior angles

pairs of angles formed on the same side of a transversal crossing two parallel lines and sum to 180°

alternate angles

pairs of angles that are formed by parallel lines cut by a transversal, positioned on opposite sides of the transversal

corresponding angles

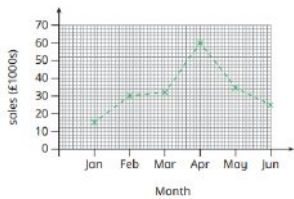
pairs of angles that are in the same relative position when parallel lines are cut by a transversal



Summer Block 2

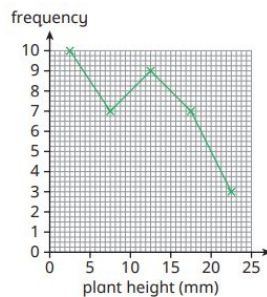
Graphs and diagrams

Higher Tier



Time series graphs track a variable over time. Time is always plotted on the x-axis. We use them to analyse trends over time.

Frequency polygons show which group of values are most and least popular. We plot the midpoint of the class intervals on the x-axis and frequency on the y-axis.

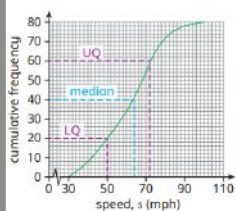
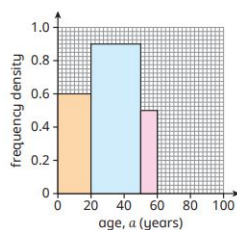


stem	leaf
0	1 4 7 9
1	3 5 8
2	2 6 8
3	1 4 7
4	2 9

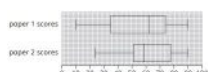
Key
2 | 6 = 26

Stem and leaf diagrams are a simple visual way of displaying a data set's distribution. Values must be listed in ascending order. Typically, 10s go in the stem and units go in the leaf, but a key can be used to change this.

Histograms look like bar charts with unequal bar widths. We calculate frequency density for each interval by dividing the frequency by the class width. The frequency of each group is represented by the bar's area.



Cumulative frequency diagrams show the distribution for continuous data. Quartiles and the median can be calculated by finding fractions of the cumulative frequency.



Box plots clearly display low values, high values, quartiles, and the median.

Step 1	Pie charts
Step 2	Time series graphs
Step 3	Frequency polygons
Step 4	Stem-and-leaf diagrams
Step 5	Draw histograms
Step 6	Interpret histograms
Step 7	Draw cumulative frequency diagrams
Step 8	Interpret cumulative frequency diagrams
Step 9	Box plots
Step 10	Compare distributions using box plots (E)

Key vocabulary

trend	general direction or pattern that the data points follow
time series graph	visual representation of data points plotted over time to show trends, patterns, or changes
frequency polygon	line graph joining points plotted at class midpoints and their frequencies
stem	part of a stem-and-leaf diagram that shows the leading place value of each number
leaf	final digit that shows individual data values within each stem group
median	type of average found by identifying the middle number in an ordered list
mode	type of average found by identifying the item that has the greatest frequency in a data set
range	difference between the greatest and smallest pieces of data in a set
continuous data	measured data that can take any value, including decimals
histogram	diagram representing continuous data using bars whose area is proportional to frequency
frequency density	value calculated by dividing frequency by class width
cumulative frequency	running total of frequencies, showing the number of data points less than or equal to each class interval
upper quartile	value below which 75% of the data lies
lower quartile	value below which 25% of the data lies
interquartile range	difference between the upper quartile and lower quartile
box plot	diagram that shows a distribution using five key values



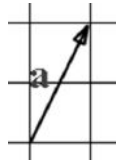
Summer Block 3

Vectors

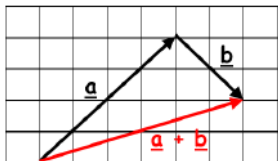
Higher Tier

Vectors are a way of describing a **movement**. They have a horizontal and vertical component: the top number represents movement left (-) or right (+), while the bottom number represents movement up (+) or down (-). See below for some examples of how we can show a vector.

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ or } \overrightarrow{AB} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

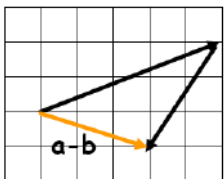


We can **add vectors** using two different methods. Look below at the column method and the diagram method. Which do you prefer?



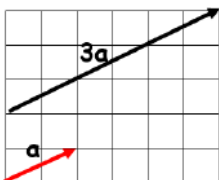
$$\begin{aligned} \mathbf{a} + \mathbf{b} &= \\ \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} &= \\ \begin{pmatrix} 6 \\ 2 \end{pmatrix} \end{aligned}$$

To **subtract vectors**, we can use similar methods to addition. For the drawn method, we change the direction of the second vector before adding.



$$\begin{aligned} \mathbf{a} - \mathbf{b} &= \\ \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} &= \\ \begin{pmatrix} 3 \\ -1 \end{pmatrix} \end{aligned}$$

We can **multiply vectors by a scalar** (a number). This works in a similar way to an enlargement; each value gets multiplied by the scalar.



$$3\mathbf{a} = 3\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

Step 1

Understand and represent vectors

Step 2

Vector notation

Step 3

Vectors multiplied by a scalar

Step 4

Add vectors

Step 5

Add and subtract vectors

Step 6

Vector journeys in shapes

Step 7

Vectors in quadrilaterals

Step 8

Parallel vectors

Key vocabulary

direction

general way in which something is facing or pointing

magnitude

length or size of a vector regardless of direction

column vector

vector represented numerically where the top and bottom values represent horizontal and vertical direction respectively

parallel vectors

two or more vectors that are multiples of each other

resultant vector

single vector that has the same effect as two or more vectors acting together

scalar

quantity representing magnitude but not direction

vector journey

route traced around a shape represented by the addition of vectors



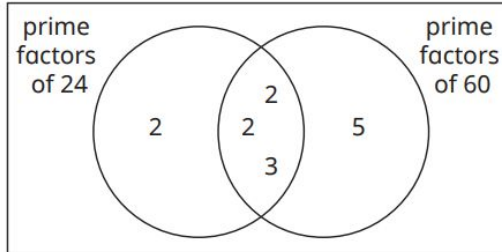
Summer Block 4

Factors, powers and surds

Higher Tier

$$24 = 2 \times 2 \times 2 \times 3$$

$$60 = 2 \times 2 \times 3 \times 5$$



Lowest common multiple = $2 \times 2 \times 2 \times 3 \times 5 = 120$

Highest common factor = $2 \times 2 \times 3 = 12$

In a **fractional power**, the numerator acts like an ordinary index. The denominator acts like a root.

$$a^{\frac{1}{2}} = \sqrt{a}$$

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

$$a^{\frac{3}{4}} = (\sqrt[4]{a})^3$$

In a **negative power**, the negative sign tells us to find the reciprocal of the positive power.

$$b^{-3} = \frac{1}{b^3}$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

$$\sqrt{50} = \sqrt{25} \sqrt{2} = 5\sqrt{2}$$

To **simplify** a value given as a square root, look for a square factor. We can use the factor to split our root into the product of two roots. Finally, find the square root of the square factor.

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

To **rationalise** a denominator means to get rid of any irrational numbers in the denominator e.g. surds. In the example above, we do so by multiplying both the numerator and denominator by the term in or numerator. In the example below, we need to use the **conjugate** of our denominator instead.

$$\frac{5}{2 + \sqrt{3}} = \frac{5}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{10 - 5\sqrt{3}}{4 - 2\sqrt{3} + 2\sqrt{3} - 3}$$

$$\frac{10 - 5\sqrt{3}}{4 - 2\sqrt{3} + 2\sqrt{3} - 3} = \frac{10 - 5\sqrt{3}}{7}$$

Step 1 Prime factorisation, HCF and LCM

Step 2 Powers, roots and negative indices

Step 3 Fractional indices

Step 4 Four operations with surds

Step 5 Simplify surds

Step 6 Expand single brackets with surds

Step 7 Rationalise the denominator

Step 8 Expand double brackets with surds

Step 9 Rationalise the denominator with more complex denominators (E)

Step 10 Solve problems with surds

Key vocabulary

highest common factor common factor of two or more integers or terms that has the greatest value or power

lowest common multiple lowest multiple shared by two or more numbers

prime factor factor of a number that is prime

base number that is multiplied when using an index

index small number to the right and above the base number or variable that shows how many times the base is multiplied by itself

power way to express repeated multiplication of a number using a base and an index

root the root of a number is the value that when raised to a specific power gives the number

reciprocal one of a pair of numbers whose product is equal to 1

surd irrational number expressed as the root of a positive integer

simplify write an expression in a simpler equivalent form

evaluate to work out the numerical value

rationalise rewrite an irrational expression so that the denominator is a rational number

conjugate one of a pair of binomials that differ only by the sign between their terms, for example the conjugate of $a + b$ is $a - b$



Summer Block 5

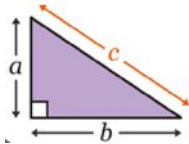
Pythagoras and trigonometry

Higher Tier

Pythagoras' Theorem can be used to find a missing side length in a right-angle triangle when we know the other

$$a^2 + b^2 = c^2 \text{ (finding the hypotenuse)}$$

$$c^2 - b^2 = a^2 \text{ (finding another side)}$$



The **Sine**, **Cosine**, and **Tangent** functions can be used to calculate missing side lengths in right-angle triangles when we know one other angle and one side.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

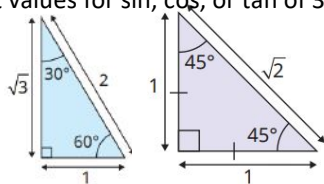
$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

To find a missing angle in a right-angle triangle, where we know two sides, we can utilise the **inverse Sine**, **Cosine**, and **Tangent** functions.

$$\theta = \sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) \quad \theta = \cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)$$

$$\theta = \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$$

We can utilise these triangles to help us recall the exact values for sin, cos, or tan of 30, 45, and 60.



In **non-right-angle triangles**, we can't use the above formulae. Instead, we consider using the formulae below.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{Sine rule (side)}$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \quad \text{Sine rule (angle)}$$

$$\text{Cosine rule (side)} \quad a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$\text{Cosine rule (angle)} \quad A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\text{Area} = \frac{1}{2}ab \sin(C) \quad \text{Area of a triangle}$$

Step 1 Pythagoras' theorem (find any side)

Step 2 Use trigonometric ratios to find an unknown side length

Step 3 Use trigonometric ratios to find an unknown angle

Step 4 Exact trigonometric values

Step 5 Trigonometry in 3-D shapes (E)

Step 6 Area of a non-right-angled triangle

Step 7 Use the sine rule to find an unknown length

Step 8 Use the sine rule to find an unknown angle

Step 9 Use the cosine rule to find an unknown length

Step 10 Use the cosine rule to find an unknown angle

Key vocabulary

hypotenuse longest side of a right-angled triangle that subtends the right angle

Pythagoras' theorem rule that states that, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

opposite side of a right-angled triangle that subtends a given angle

adjacent side of a right-angled triangle that forms a given angle with the hypotenuse

dimensions measurements, such as length, width and height, that describe the size and extent of an object

plane flat, two-dimensional surface that extends infinitely in all directions

perpendicular where lines, planes or surfaces meet at a right angle

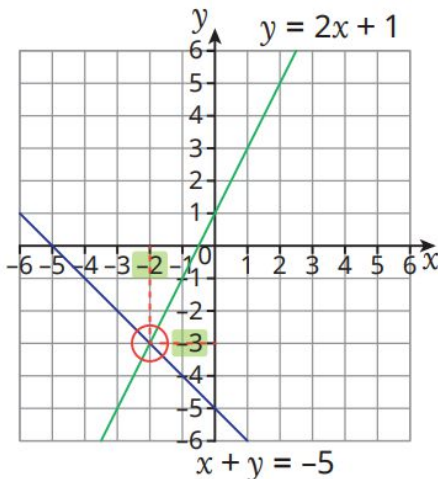


Summer Block 6

Simultaneous equations

Higher Tier

We can solve simultaneous equations using a graph. These lines intersect at $(-2, -3)$. We can write the solutions in the form: $x = -2$
 $y = -3$



Elimination is a dependable method for solving linear simultaneous equations.

$$\begin{array}{rcl} 2x + 3y & = & 39 \quad \times 2 \\ 5x - 2y & = & -7 \quad \times 3 \end{array}$$

We need one pair of coefficients to match, so we cross multiply.

$$\begin{array}{rcl} 4x + 6y & = & 78 \\ 15x - 6y & = & -21 \\ \hline 19x & = & 57 \\ x & = & 3 \end{array}$$

To eliminate our matched terms, we add (different signs) or subtract (same signs).

$$\begin{array}{rcl} 2(3) + 3y & = & 39 \\ 6 + 3y & = & 39 \\ 3y & = & 33 \quad -6 \\ y & = & 11 \quad \div 3 \end{array}$$

Once we solve for the first variable, we can substitute into any equation to solve for the other. Our final answer needs a solution for each variable.

$$x = 3, y = 11$$

Substitution helps us solve more complex simultaneous equations, including those involving non-linear equations.

Consider: $x^2 + 2y = 9$ and $x = y - 3$

How did we get the first equation in the yellow box?

What could we do with the final equation in the yellow box?

$$\begin{aligned} (y - 3)^2 + 2y &= 9 \\ y^2 - 6y + 9 + 2y &= 9 \\ y^2 - 4y &= 0 \end{aligned}$$

Step 1	Solve simultaneous equations using graphs
Step 2	Solve simultaneous equations (no adjustments)
Step 3	Solve simultaneous equations (adjust one)
Step 4	Solve simultaneous equations (adjust both)
Step 5	Solve simultaneous equations by substitution
Step 6	Solve problems with simultaneous equations
Step 7	Solve simultaneous equations (one linear, one non-linear) using graphs
Step 8	Solve simultaneous equations (one linear, one non-linear) by equating expressions (E)
Step 9	Solve simultaneous equations (one linear, one non-linear) using substitution (E)

Key vocabulary

coefficient	number in front of a variable indicating the multiple of the variable
simultaneous equations	set of two or more equations with unknowns in common, where the solutions satisfy all equations at the same time
solve	find the value(s) or set of values of an unknown in an equation or inequality
variable	symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae
unknown	value not yet known, indicated with a letter and used in an equation or inequality
linear equation	equation that forms a straight line when graphed
non-linear equation	equation that does not form a straight line when graphed
coordinates	pair of numbers that shows the horizontal and vertical position of a point
intersection	point where two graphs cross
solution	value of an unknown that satisfies an equation or inequality
quadratic	involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$



Autumn Block 1

Algebraic manipulation

Foundation Tier

Work out the value of the expression when $a = 4$, $b = 2$ and $c = 7$

$$ab + c$$

$$a \times b + c$$

$$4 \times 2 + 7 = 8 + 7$$

$$= 15$$

definition	characteristics
terms with the same variable(s) and power(s)	- Same variable(s) - Same power
examples	non-examples
x $3x$ $\frac{1}{2}x$ $\frac{x}{7}$ $-5x$	$6xy$ $4x^2$ 8^x $\frac{3}{x}$

Like terms

$$4^5 \times 4^3 = 4^8$$

$$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^8$$

$$\frac{4^5}{4^3} = 4^2$$

$$\frac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4} = 4^2$$

$$(4^5)^3 = 4^{15}$$

$$\begin{aligned}
 4^5 \times 4^5 \times 4^5 &= 4^{5+5+5} \\
 &= 4^{3 \times 5} \\
 &= 4^{15}
 \end{aligned}$$

Step 1 Substitution

Step 2 Collect like terms

Step 3 Simplify expressions

Step 4 Addition and subtraction laws for indices

Step 5 Powers of powers

Step 6 Expand a single bracket

Step 7 Factorise into a single bracket

Key vocabulary

expression collection of terms involving variables (letters) and numbers

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

formula mathematical rule or equation that shows the relationship between different variables or quantities

substitute replace letters with numerical values

like terms terms with the same variable(s) and power(s)

coefficient number in front of a variable indicating the multiple of the variable

simplify write an expression in a simpler equivalent form

power way to express repeated multiplication of a number using a base and an index

base number that is raised to a certain index, for example, 10^3 has a base of 10

index written as a small number to the right and above the base number, which indicates how many times the base has been multiplied by itself

indices plural of index

expand multiply each term inside the bracket by the expression outside the bracket

highest common factor (HCF) common factor of two or more integers or terms that has the greatest value or power



Autumn Block 2

Equations, inequalities and formulae

Foundation Tier

Solve $3m - 9 = 12$

$$3m - 9 = 12$$

$$+ 9 \quad + 9$$

$$3m = 21$$

$$\div 3 \quad \div 3$$

$$m = 7$$

Solve $5t + 3 = 3t + 17$

$$\begin{array}{r} 5t + 3 = 3t + 17 \\ -3t \quad -3t \\ \hline 2t + 3 = 17 \\ -3 \quad -3 \\ \hline 2t = 14 \\ \div 2 \quad \div 2 \\ \hline t = 7 \end{array}$$

Make b the subject of the formula.

$$A = \frac{1}{2}bh$$

$$A = \frac{bh}{2}$$

$$\times 2 \quad \times 2$$

$$2A = bh$$

$$\div h \quad \div h$$

$$\frac{2A}{h} = b$$

Step 1

Solve equations

Step 2

Solve fractional equations

Step 3

Solve equations with unknowns on both sides

Step 4

Understand inequalities

Step 5

Solve inequalities

Step 6

Change the subject of a simple formula

Step 7

Change the subject of a known formula

Step 8

Change the subject of a complex formula (E)

Key vocabulary

equation	statement to show that two expressions are equal
inverse	opposite effect of
expression	collection of terms involving variables and numbers
unknown	value not yet known, indicated with a letter and used in an equation or inequality
solution	value of an unknown that satisfies an equation or inequality
coefficient	number in front of a variable indicating the multiple of the variable
integer	whole number that can be positive or negative
inequality	mathematical statement that shows the relationship between two expressions where one is greater than, less than, or not equal to the other
satisfy	find a value that makes the inequality true when substituted into the inequality
solve	find the value(s) or set of values of an unknown in an equation or inequality
formula	mathematical rule or equation that shows the relationship between different variables or quantities.
subject	variable that is isolated on one side of the formula
variable	symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae
rearrange	adjust a formula to isolate a specific variable or simplify an expression



Autumn Block 3

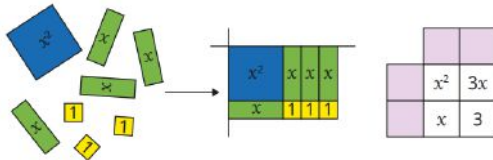
Quadratic expressions and equations

Foundation Tier

$$(1-x)(3+x) \equiv 3-2x-x^2$$



$$x^2 + 4x + 3$$

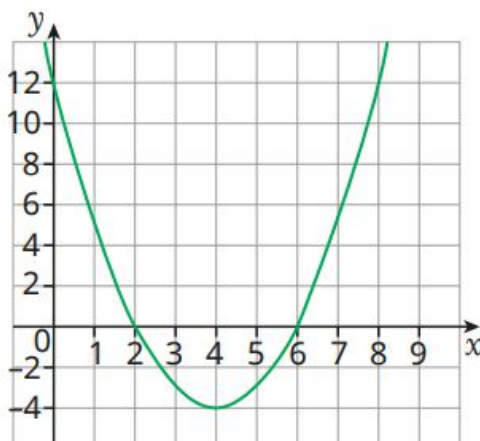


$$(x-1)(x+1) \equiv x^2 - x + x - 1 \\ \equiv x^2 - 1$$

$$\text{Solve } x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$x-2=0 \quad \text{or} \quad x-6=0$$



Step 1 Expand double brackets

Step 2 Factorise quadratic expressions (positive only)

Step 3 Factorise quadratic expressions

Step 4 Difference of two squares (E)

Step 5 Solve quadratic equations equal to 0

Step 6 Solve quadratic equations by factorisation

Step 7 Quadratic graphs of the form $y = x^2 + a$

Key vocabulary

quadratic equation or expression involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$

expand multiply each term inside the bracket by the expression outside the bracket

term number or variable or any combination of number and variable combined by multiplication or division

sum result of adding two or more numbers together

product result of a multiplication

factorise express an algebraic expression as a product of its factors using brackets; the reverse process of expanding brackets

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

expression collection of terms involving variables and numbers

solve find the value(s) or set of values of an unknown in an equation or inequality

solution value of an unknown that satisfies an equation or inequality

roots value(s) of x where the quadratic equals 0

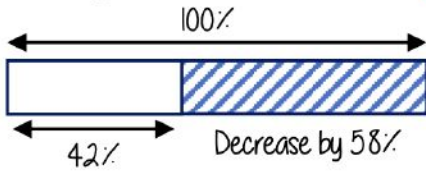


Autumn Block 4

Percentages

Foundation Tier

Percentage increase/decrease



$$100\% - 58\% = 42\%$$

$$100 - 58 = 42$$

Multiplier
Less than 1



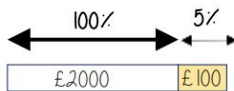
$$100\% + 12\% = 112\%$$

$$100 + 12 = 112$$

Multiplier
More than 1

Simple Interest

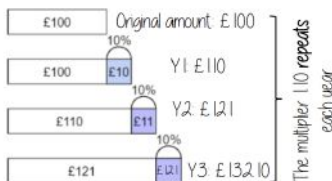
James invests
£2000 at 5%
simple interest



The original value increases
by this amount every year

Compound Interest

Tess invests
£100 at 10%
compound
interest for 3
years



Find the original value

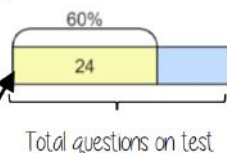
In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test?

$$\text{Original} \times 0.6 = 24$$

$$24 \div 0.6 = 40 \text{ marks}$$

$$10\% = 6$$

$$100\% = 40$$



Step 1

Percentage of an amount

Step 2

Percentage increase and decrease

Step 3

Repeated percentage change (E)

Step 4

Express one number as a fraction or percentage of another

Step 5

Express a change as a percentage

Step 6

Find the original value after a percentage change

Step 7

Simple interest

Step 8

Compound interest (E)

Step 9

Choose appropriate methods to solve percentage problems

Key vocabulary

percentage expressing a part of a whole as a number out of 100

decimal number number with a whole and fractional part, for example 3.2, 5.43

multiplier number that is multiplied with another number to increase or decrease its value

depreciate decrease in value over time

fraction number with a numerator and a denominator that represents a proportion

equivalent fraction fraction that represents the same value as another fraction with different numerators and denominators

percentage change how much a quantity has increased or decreased relative to its original value, calculated as a percentage

profit positive difference between the selling price and the cost price of an item, indicating financial gain

loss negative difference between the cost price and the selling price of an item, indicating a financial loss

simple interest interest calculated on the principal amount and then multiplied by the time period of the investment or loan

deposit money placed in a bank account or given as an initial payment for a service or purchase

investment amount deposited in a savings account with the goal of earning a financial gain or increasing the value over time

per annum "per year", used to describe interest rates or payments that occur annually

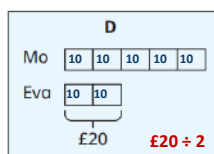
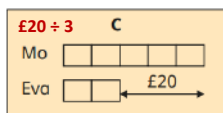
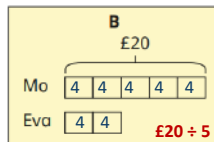
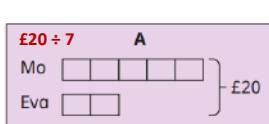
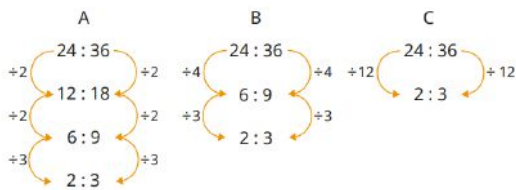
compound interest interest calculated on both the principal amount and the accumulated interest from previous periods



Autumn Block 5

Ratio and scale

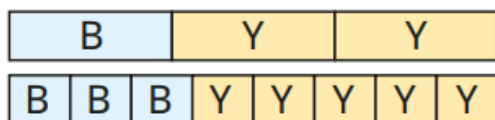
Foundation Tier



Max and Dexter make green paint by mixing blue and yellow paint together.

Max
blue : yellow = 1 : 2

Dexter
blue : yellow = 3 : 5



Who makes a darker shade of green paint?

Red	Blue	Yellow
R R	B B B	
	B B B	Y

R : B : Y
2 : 3
6 : 1

Red	Blue	Yellow
R R	B B B	
R R R	B B B B	Y

R : B : Y
2 : 3
4 : 6 : 1

The ratio of x to y is 2 : 3
 $x + y = 30$
Work out the values of x and y.

$x = 6$ $y = 6$ } 30

$x = 6$ $y = 6$ $y = 6$ } 30

$30 \div 5 = 6$

$x = 2 \times 6$ $y = 3 \times 6$
 $x = 12$ $y = 18$

Step 1

Equivalent ratios

Step 2

Share in a ratio (whole given)

Step 3

Share in a ratio (part or difference given)

Step 4

Link ratios and fractions

Step 5

Combine a set of ratios (E)

Step 6

Share in a ratio (algebraically) (E)

Step 7

Ratios and scales

Key vocabulary

highest common factor common factor of two or more integers or terms that has the greatest value or power

simplest form when the highest common factor of the values in a ratio is 1

equivalent ratio two ratios are equivalent if they simplify to the same ratio

ratio comparison of two or more values

fraction number with a numerator and a denominator that represents a proportion

numerator top number in a fraction that indicates the number of equal parts out of a whole

denominator bottom number in a fraction that indicates the number of equal parts a whole has been divided into

lowest common multiple lowest multiple shared by two or more numbers

scaling multiplication of each value by a constant to proportionally increase or decrease the size

common part same part represented in two ratios, for example, in the ratios red : blue and blue : green, the common part is blue

algebraic related to algebra, often involving expressions, equations, or symbols representing numbers or quantities

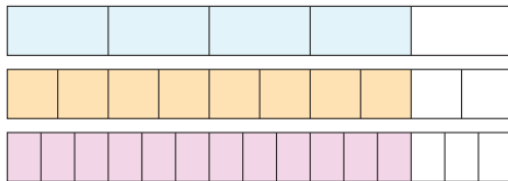
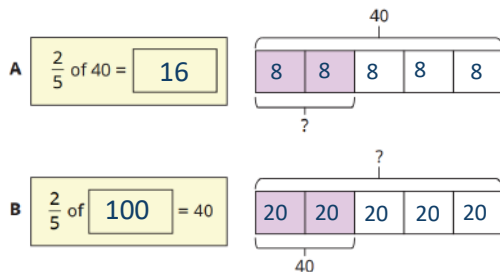
expression collection of terms involving variables (letters) and numbers



Autumn Block 6

Work with fractions

Foundation Tier



$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

Method A $2\frac{2}{5} \times 3\frac{8}{9} = \frac{12}{5} \times \frac{35}{9} = \frac{420}{45} = 9\frac{15}{45} = 9\frac{1}{3}$

Method B $2\frac{2}{5} \times 3\frac{8}{9} = \frac{12}{5} \times \frac{35}{9} = \frac{4}{1} \times \frac{7}{3} = \frac{28}{3} = 9\frac{1}{3}$

$$6\frac{4}{5} \div 2\frac{2}{5} = \frac{34}{5} \div \frac{12}{5}$$

Write the mixed numbers as improper fractions.

$$\frac{34}{5} \div \frac{12}{5} = \frac{34}{5} \times \frac{5}{12}$$

Rewrite the division as a multiplication by the reciprocal of the divisor.

$$\frac{34}{5} \times \frac{5}{12} = \frac{17}{1} \times \frac{1}{6}$$

Look for common factors between the numerators and denominators of each fraction.

$$\frac{17}{1} \times \frac{1}{6} = \frac{17}{6} = 2\frac{5}{6}$$

Multiply the fractions and write as a mixed number.

Step 1

Fraction of an amount

Step 2

Increase or decrease an amount by a fraction

Step 3

Use a fraction to find the whole

Step 4

Equivalent fractions and mixed numbers

Step 5

Add and subtract fractions

Step 6

Multiply fractions

Step 7

Divide fractions

Step 8

Solve problems with fractions

Key vocabulary

fraction number with a numerator and a denominator that represents a proportion

numerator top number in a fraction that indicates the number of equal parts out of a whole

denominator bottom number in a fraction that indicates the number of equal parts a whole has been divided into

whole total or complete amount

increase make a number greater

decrease make a number smaller

equivalent fraction fraction that represents the same value as another fraction with different numerators and denominators

mixed number integer and a proper fraction represented together to form a number

improper fraction fraction where the numerator is greater than the denominator

simplify write an equivalent fraction by dividing the numerator and denominator by a common factor

product result of a multiplication

reciprocal one of a pair of numbers whose product is equal to 1

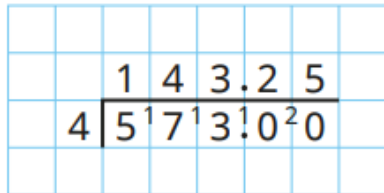
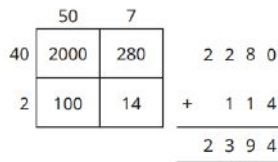
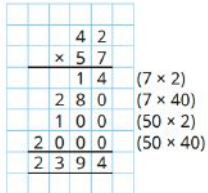
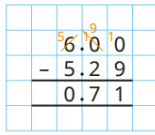


Spring Block 1

Non-calculator methods

Foundation Tier

$$6 - 5.29$$



$$3 \times 8 = 24$$

$$8 \times 3 = 24$$

$$24 \div 3 = 8$$

$$24 \div 8 = 3$$

$$-3 \times 8 = -24$$

$$8 \times -3 = -24$$

$$-24 \div -3 = 8$$

$$-24 \div 8 = -3$$

$$-3 \times -8 = 24$$

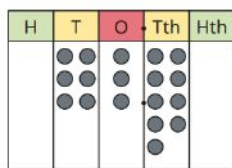
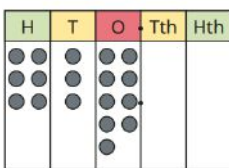
$$-8 \times -3 = 24$$

$$24 \div -3 = -8$$

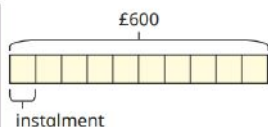
$$24 \div -8 = -3$$

$$3 \times 213 = 639$$

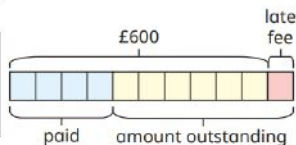
$$3 \times 21.3 = 63.9$$



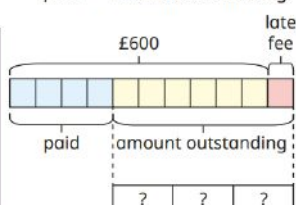
Mr Fisher buys a laptop for £600. He plans to pay in 10 equal instalments.



He pays the 4th instalment late and gets a £15 late fee.



He wants to pay the remaining amount plus the late fee in 3 equal instalments. How much money must he pay per instalment?



Step 1

Place value for integers and decimals

Step 2

Compare and order numbers

Step 3

Add and subtract integers and decimals

Step 4

Multiply and divide integers and decimals

Step 5

Four operations with directed number

Step 6

Order of operations

Step 7

Related calculations

Step 8

Solve multi-step problems

Key vocabulary

ascending order ordered from smallest to greatest

descending order ordered from greatest to smallest

product result of a multiplication

sum result of adding two or more numbers together

inverse opposite effect of

deposit money placed in a bank account or given as an initial payment for a service or purchase

instalments regular, smaller payments made over time to pay back a greater amount of money

profit positive difference between the selling price and the cost price of an item, indicating financial gain

interest amount of money charged or earned based on a percentage of the original amount over a period of time



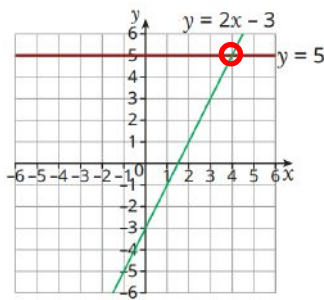
Spring Block 2

Straight line graphs

Foundation Tier

$x + y = 5$	→	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td></tr></table>	x	-2	-1	0	1	2	y	7	6	5	4	3
x	-2	-1	0	1	2									
y	7	6	5	4	3									
$y = 2x - 3$	→	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>-7</td><td>-5</td><td>-3</td><td>-1</td><td>1</td></tr></table>	x	-2	-1	0	1	2	y	-7	-5	-3	-1	1
x	-2	-1	0	1	2									
y	-7	-5	-3	-1	1									
$y = x + 7$	→	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	x	-2	-1	0	1	2	y	5	6	7	8	9
x	-2	-1	0	1	2									
y	5	6	7	8	9									

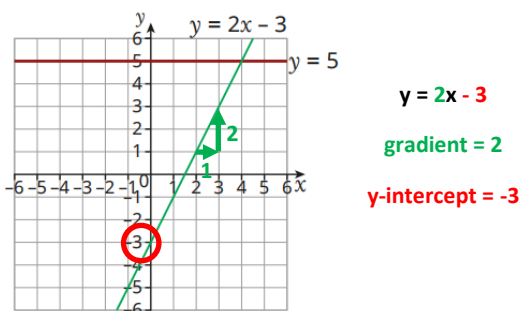
How can we generate these values from the equations shown?



The lines $y = 2x - 3$ and $y = 5$ meet at (4, 5).

The equation $2x - 3 = 5$ has a solution of $x = 4$.

Is there a link between these statements?



Step 1

Draw straight line graphs

Step 2

Find solutions to equations using straight lines graphs

Step 3

Explore gradients

Step 4

$y = mx + c$

Step 5

Find the equation of a line from a graph (E)

Step 6

Find the midpoint of a line segment

Step 7

Equation of a straight-line graph given one point and a gradient

Step 8

Equation of a straight-line graph given two points (E)

Step 9

Real-life straight-line graphs

Key vocabulary

linear

where a relationship or pattern changes at a constant rate, forming a straight line when graphed

solution

value of an unknown that satisfies an equation or inequality

gradient

measure of how steep a line is, showing the rate of change between two points on a graph

y-intercept

point where a line or curve crosses the y-axis on a graph

parallel lines

lines that maintain a constant distance from each other

midpoint

halfway between two intervals on a number line or two numbers

line segment

part of a line between two points

mean

type of average found by dividing the total of the values by the number of values in the set



Spring Block 3

Probability

Foundation Tier

Colour	red	blue	green
Probability	0.3	0.2	

Probabilities sum to 1, so:

$$P(\text{green}) = 1 - (0.3 + 0.2) = 0.5$$

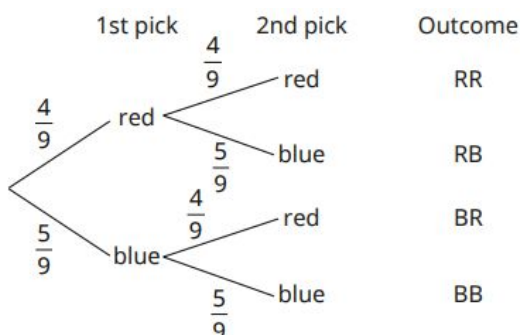
Relative frequency =

$\frac{\text{number of successes}}{\text{number of trials}}$

A		B					
(T, 1) (T, 2)		Dice					
(T, 3) (T, 4)							
(T, 5) (T, 6)							
(H, 1) (H, 2)		1	2	3	4	5	6
Coin	Heads	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
	Tails	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)
(H, 3) (H, 4)							
(H, 5) (H, 6)							

A and B are examples of sample space diagrams for rolling a dice and flipping a coin.

A bag contains 4 red and 5 blue counters. A counter is selected at random, its colour is noted and then it is replaced. Then another counter is selected.



Step 1

Probability of a single event

Step 2

Use the property that probabilities sum to 1

Step 3

List outcomes

Step 4

Relative frequency

Step 5

Sample spaces for 1 or more events

Step 6

Two-way tables

Step 7

Frequency trees

Step 8

Independent events

Step 9

Tree diagrams for independent events

Step 10

Tree diagrams for dependent events (E)

Key vocabulary

probability

measure of how likely an event is to happen

event

outcome or group of outcomes that have a probability of happening

outcome

possible result of an experiment or trial

experimental probability

likelihood of an event based on results from trials

theoretical probability

expected likelihood of an event happening

frequency

number of times an outcome appears in a dataset

expected frequency

number of times an event is predicted to occur, based on its theoretical probability or relative frequency

relative frequency

number of times an event occurs compared to the total number of trials

sample space

complete set of all possible outcomes of a probability experiment

independent

one event does not affect the probability of the other event occurring

dependent

one event affects the probability of the other event occurring



Spring Block 4

Rounding and estimation

Foundation Tier

Round 5.63705
to one decimal place.

5.6|3705 Keep 1 digit after the decimal point

5.6|[↓]3705 The next digit tells us to round down, because it's less than 5

5.6 We remove the rest of the digits, as we only want 1 decimal place

Round 36,271 to three
significant figures.

36,2|71 Keep 3 digits from the first significant figure (non-zero digit)

36,3|[↑]71 The next digit tells us to round up, because it's 5 or greater

36,300 We replace the rest of the digits with zeros, as they affect the value of the number

Work out an estimate for the value of 54×9.2

A	B	C	D
54×9	50×10	50×9	55×10

An estimate finds an approximate answer to a question. We should round to 1 significant figure to find an estimate in most cases.

Error intervals

$x = 3.4$ rounded to 1 decimal place	$3.4 \leq x < 3.5$
$x = 3.44$ rounded to 3 significant figures	$3.35 \leq x < 3.45$
$x = 3.4$ truncated to 1 decimal place	$3.435 \leq x < 3.445$

Step 1

Round to decimal places

Step 2

Round to significant figures

Step 3

Estimate answers to calculations

Step 4

Use of a calculator

Step 5

Error intervals (including truncation) (E)

Key vocabulary

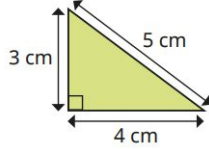
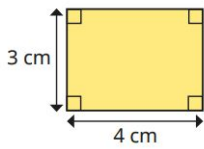
round	replacing a number with an approximate value
significant figures	digits in a number starting from the first non-zero digit
decimal place	position of a digit to the right of the decimal point
degree of accuracy	indication of how precise a number is
estimate	an appropriate value
approximate	as close to the actual value as possible but not completely accurate
overestimate	an estimate that is greater than the actual value
underestimate	an estimate that is less than the actual value
truncate	shorten a number by removing digits beyond a certain point without rounding
limits	minimum and maximum values, often stated with inequalities to indicate whether the limit can be included
error interval	range of values a number could be, to a degree of accuracy



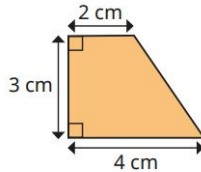
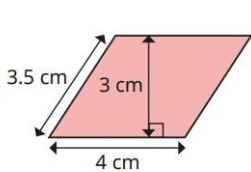
Spring Block 5

Perimeter, area and volume

Foundation Tier

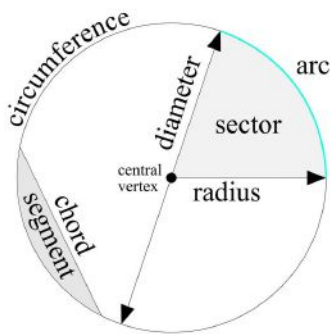


$$\text{area} = \text{length} \times \text{width} \quad \text{area} = \text{base} \times \text{height} \div 2$$



$$\text{area} = \text{base} \times \text{height}$$

$$\text{area} = \frac{a+b}{2} \times h$$



$$\text{Area} = \pi \times \text{radius}^2$$

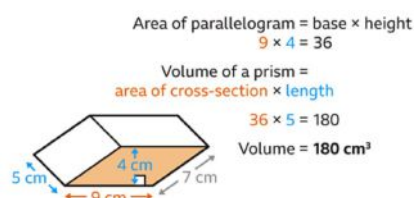
$$A = \pi \times r^2$$

$$\text{Circumference} = \pi \times \text{Diameter}$$

$$C = \pi \times D$$

The **surface area** of a shape is the sum of the areas of each face. Don't forget about faces you can't see!

$$\text{Volume of a prism} = \text{cross-sectional area} \times \text{depth}$$



Step 1 Name 2-D and 3-D shapes

Step 2 Perimeter of a 2-D shape

Step 3 Area of a 2-D shape

Step 4 Area of a compound shape

Step 5 Recognise and label parts of a circle

Step 6 Circumference of a circle

Step 7 Area of a circle

Step 8 Volume of a prism

Step 9 Nets

Step 10 Surface area of a prism

Key vocabulary

radius	distance from the centre of a circle to any point of the circumference
diameter	straight line passing through the centre of a circle from one point on the circumference to another
arc	proportion of the circumference of a circle
sector	part of a circle formed from two radii subtended by an arc
chord	line segment that connects any two points on the circle's circumference
segment	region enclosed by a chord and its arc
perpendicular height	height of a shape, measured at a right angle from the base to a vertex or edge
face	flat surface on a 3-D shape
edge	straight line between two vertices or faces of a shape
vertex	point at which two or more edges (3-D) or sides (2-D) meet



Spring Block 6

Interpret and represent data

Foundation Tier

Averages & range from a list:

3, 3, 5, 7, 11, 15

$$\text{Mean} = (3+3+5+7+11+15) \div 6 = 7.3 \text{ (1dp)}$$

$$\text{Median} = (5 + 7) \div 2 = 6$$

$$\text{Mode} = 3 \text{ (most common value)}$$

$$\text{Range} = 15 - 3 = 12$$

Averages from a frequency table:

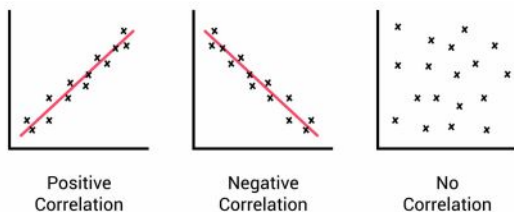
Test score, s	Frequency	Cumulative frequency	fx
40 ≤ s < 50	45	8	45 × 8 = 360
50 ≤ s < 60	55	13	55 × 5 = 275
60 ≤ s < 70	65	24	65 × 11 = 715
70 ≤ s < 80	75	30	75 × 6 = 450
	<u>30</u>		<u>1800</u>

$$\text{Estimated Mean} = \frac{\sum fx}{\sum f} = \frac{1800}{30} = 60$$

Modal class = $60 \leq s < 70$
(the interval with the highest frequency)

$$\text{Class containing median} = \frac{\sum f + 1}{2} \text{th value}$$

$$= \frac{30 + 1}{2} = 15.5 \text{th value} = 60 \leq s < 70$$



Key vocabulary

population	whole group that the data is collected from or analysed in a statistical investigation
sample	smaller group taken from a population to represent the whole in a statistical study
stratified sample	sample taken by dividing the population into groups and selecting a random sample from each group in proportion to its size
census	survey that collects data from every member of a population
random sample	group chosen from a population where every member has an equal chance of being selected
interpolation	estimating a value within the range of known data points
extrapolation	estimating a value beyond the range of known data points

Step 1

Averages and range

Step 2

Averages from an ungrouped frequency table

Step 3

Mean from a grouped frequency table

Step 4

Averages from a grouped frequency table

Step 5

Use data to compare distributions (E)

Step 6

Types of data

Step 7

Sampling

Step 8

Scatter graphs

Step 9

Interpolation and extrapolation (E)

Key vocabulary

average	number or item representing the typical value of a set of data
mean	type of average found by dividing the total of the values by the number of values in the set
median	type of average found by identifying the middle number in an ordered list
mode	type of average found by identifying the item that has the greatest frequency in a data set
range	difference between the greatest and smallest pieces of data in a set
frequency	number of times a value appears in a data set
modal class	class interval with the greatest frequency
cumulative frequency	running total of frequencies, showing the number of data points less than or equal to each class interval
discrete data	data that can be counted
continuous data	measured data that can take any value, including decimals
qualitative data	non-numerical data
primary data	data collected by the researcher through methods like surveys, experiments or observations
secondary data	data collected by someone else, such as from websites, reports or books

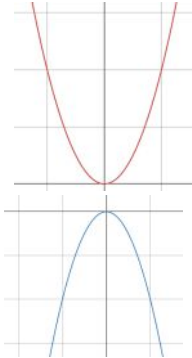


Spring Block 7

Non-linear graphs

Foundation Tier

Quadratic graphs have a U or n-shaped curve. This shape is called a parabola.

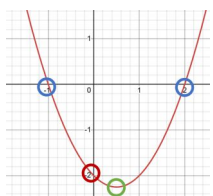


We can tell an expression or equation is quadratic if it has an x^2 term and no higher powers of x .

The **root(s)** of a quadratic graph are the point(s) where the graph intercepts the x-axis.

The **y-intercept** is where the graph intercepts the y-axis.

The **turning point** is where the graph changes from having a positive gradient to a negative gradient (or vice-versa).



Roots: $x = -1$ and $x = 2$
y-intercept: $(0, -2)$
Turning point: $(0.5, -2.25)$

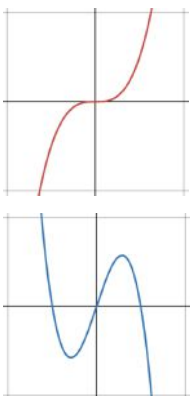
$$y = x^2 - x - 2$$

Can you see the y-intercept in the equation, too?

$$(x - 2)(x + 1) = 0$$

What about the roots?

To **solve** a quadratic graph of the form $ax^2 + bx + c = k$, look for where it intercepts the line $y = k$.



Cubic graphs have a unique shape. They go from low values of y to high values of y , or vice versa. Cubic graphs can have one or two turning points.

We can tell an expression or equation is quadratic if it has an x^3 term and no higher powers of x .

Step 1

Quadratic graphs

Step 2

Intercepts and roots of quadratic graphs (E)

Step 3

Cubic graphs

Step 4

Approximate solutions to equations using graphs (E)

Key vocabulary

quadratic	involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$
parabola	approximately U-shaped symmetrical curve that often represents the graph of a quadratic function
substitute	replace letters with numerical values
intercept	point where a line or curve crosses an axis on a graph
root	value(s) of x where the quadratic equals 0
solution	value of an unknown that satisfies an equation or inequality
cubic	involving the third (and no higher) power of a variable or unknown, typically in the form $ax^3 + bx^2 + cx + d$
estimate	number or result that is not exact, but is suitably close to the actual value
turning point	point where the gradient of a graph changes from positive to negative or vice versa
solution	value of an unknown that satisfies an equation or inequality
root	value(s) of x where the quadratic equals 0
approximate	as close to the actual as possible but not completely accurate
intersect	where straight lines or curves meet and cross at a point



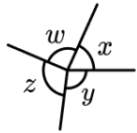
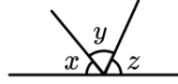
Summer Block 1

Angles

Foundation Tier

The sum of angles on a straight line is 180° .

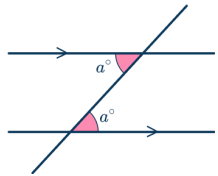
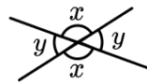
$$x + y + z = 180$$



The sum of angles at a point is 360° .

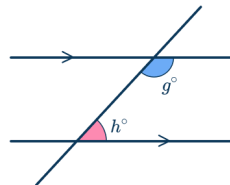
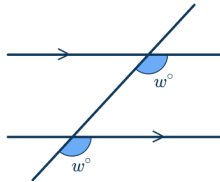
$$w + x + y + z = 360$$

Vertically opposite angles are equal in size.



Alternate angles are equal (z-shape).

Corresponding angles are equal (f-shape).



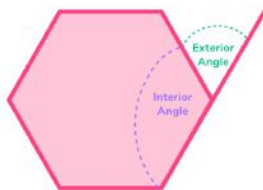
Co-interior angles add to 180° (c-shape).

Interior angles in a triangle add to 180° .



Interior angles in a quadrilateral add to 360° .

An adjacent interior and exterior angle add to 180° .



Exterior angles in any polygon add to 360° .

$$\text{Sum of interior angles} = (n - 2) \times 180^\circ$$

Step 1

Angles around a point, on a straight line and vertically opposite

Step 2

Angles in a triangle

Step 3

Angles in a quadrilateral

Step 4

Exterior angles of a polygon

Step 5

Interior angles of a polygon

Step 6

Solve problems with angles in polygons (E)

Step 7

Alternate and corresponding angles

Step 8

Co-interior, alternate and corresponding angles

Step 9

Prove geometric facts (E)

Key vocabulary

vertically opposite

pair of equal angles that are opposite each other at a vertex created by two straight intersecting lines

triangle

polygon with three straight sides and three interior angles that sum to 180°

quadrilateral

polygon with four straight sides and four angles that sum to 360°

isosceles triangle

triangle with two equal sides and angles

right angle

angle that measures 90° and can be marked by a square marker

exterior angle

angle formed when one side of a polygon is extended, creating an angle outside the polygon

interior angle

angle inside a polygon where two sides meet

polygon

closed 2-D shape made of straight line segments

regular polygon

polygon where all the sides are equal in length and all the interior angles are equal

parallel lines

lines that maintain a constant distance from each other

transversal

line that crosses two or more other lines at distinct points, often creating pairs of angles

co-interior angles

pairs of angles formed on the same side of a transversal crossing two parallel lines and sum to 180°

alternate angles

pairs of angles that are formed by parallel lines cut by a transversal, positioned on opposite sides of the transversal

corresponding angles

pairs of angles that are in the same relative position when parallel lines are cut by a transversal



Summer Block 2

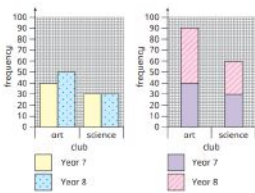
Graphs and diagrams

Foundation Tier

Pictograms are a simple way to represent frequencies for different categories. They need a key to show what each symbol represents.

Day	Number of T-shirts sold
Monday	●
Tuesday	● ● ●
Wednesday	● ●
Thursday	
Friday	● ●

Key:
● = 4

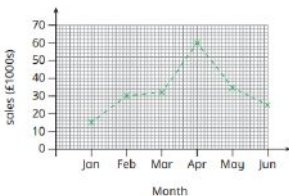


Bar charts are a more precise way to show frequencies for categories. Dual bar charts show comparative data side by side. Composite bar charts show stack up comparative data.

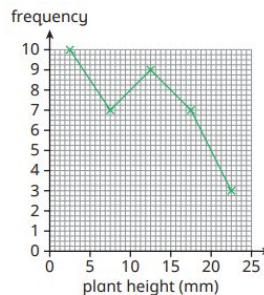
Pie charts are useful for comparing the frequencies of different groups quickly.



Time series graphs track a variable over time. Time is always plotted on the x-axis. We use them to analyse trends over time.



Frequency polygons show which group of values are most and least popular. We plot the midpoint of the class intervals on the x-axis and frequency on the y-axis.



stem	leaf
0	1 4 7 9
1	3 5 8
2	2 6 8
3	1 4 7
4	2 9

Key
2 | 6 = 26

Stem and leaf diagrams are a simple visual way of displaying a data set's distribution. Values must be listed in ascending order. Typically, 10s go in the stem and units go in the leaf, but a key can be used to change this.

Step 1 Pictograms

Step 2 Line and bar charts

Step 3 Dual and composite bar charts

Step 4 Draw pie charts

Step 5 Interpret pie charts

Step 6 Time series graphs

Step 7 Frequency polygons

Step 8 Stem-and-leaf diagrams

Key vocabulary

pictogram	way to show information using symbols that represent a certain value
key	information that shows what a symbol represents
frequency	number of times a value appears in a dataset
bar chart	chart that uses rectangular bars to represent frequencies from a data set
vertical line chart	chart that uses straight, vertical lines to represent frequencies from a data set
dual bar chart	representation that uses bars with proportional heights to show information about two or more variables
composite bar chart	chart that represents multiple data sets within a single bar, using segments to represent different categories
pie chart	circular representation of data split into sectors that represent different values
trend	general direction or pattern that the data points follow
time series graph	visual representation of data points plotted over time to show trends, patterns, or changes
frequency polygon	line graph joining points plotted at class midpoints and their frequencies
stem	part of a stem-and-leaf diagram that shows the leading place value of each number
leaf	final digit that shows individual data values within each stem group
median	type of average found by identifying the middle number in an ordered list
mode	type of average found by identifying the item that has the greatest frequency in a data set
range	difference between the greatest and smallest pieces of data in a set



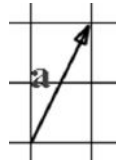
Summer Block 3

Vectors

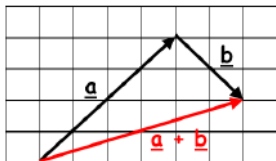
Foundation Tier

Vectors are a way of describing a **movement**. They have a horizontal and vertical component: the top number represents movement left (-) or right (+), while the bottom number represents movement up (+) or down (-). See below for some examples of how we can show a vector.

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ or } \overrightarrow{AB} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

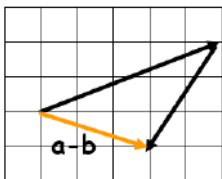


We can **add vectors** using two different methods. Look below at the column method and the diagram method. Which do you prefer?



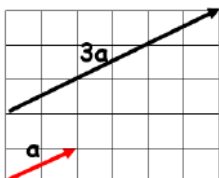
$$\begin{aligned} \mathbf{a} + \mathbf{b} &= \\ \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} &= \\ \begin{pmatrix} 6 \\ 2 \end{pmatrix} \end{aligned}$$

To **subtract vectors**, we can use similar methods to addition. For the drawn method, we change the direction of the second vector before adding.



$$\begin{aligned} \mathbf{a} - \mathbf{b} &= \\ \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} &= \\ \begin{pmatrix} 3 \\ -1 \end{pmatrix} \end{aligned}$$

We can **multiply vectors by a scalar** (a number). This works in a similar way to an enlargement; each value gets multiplied by the scalar.



$$3\mathbf{a} = 3\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

Step 1

Understand and represent vectors

Step 2

Vector notation

Step 3

Vectors multiplied by a scalar

Step 4

Add vectors

Step 5

Add and subtract vectors

Step 6

Solve problems with vectors (E)

Key vocabulary

direction

general way in which something is facing or pointing

magnitude

length or size of a vector regardless of direction

column vector

vector represented numerically where the top and bottom values represent horizontal and vertical direction respectively

parallel vectors

two or more vectors that are multiples of each other

resultant vector

single vector that has the same effect as two or more vectors acting together

scalar

quantity representing magnitude but not direction

vector journey

route traced around a shape represented by the addition of vectors

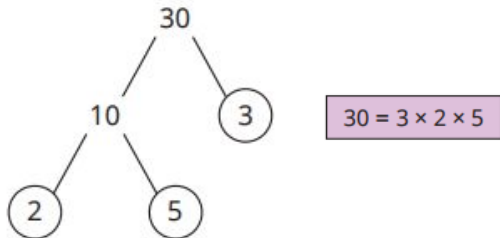


Summer Block 4

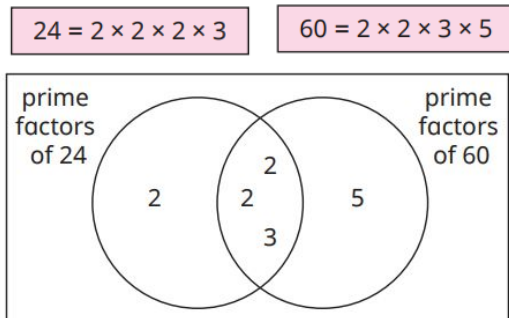
Factors, powers and surds

Foundation Tier

Prime factor decomposition means writing a number as a product of its prime factors. We can find them using repeated division.



Once we have the prime factorisation of two numbers, we can find their **lowest common multiple** or **highest common factor** using a Venn diagram. The **LCM** is the product of all numbers in the diagram. The **HCF** is the product of only the numbers in the middle section.



Lowest common multiple = $2 \times 2 \times 2 \times 3 \times 5 = 120$

Highest common factor = $2 \times 2 \times 3 = 12$

A **root** is the inverse of a power. So from the powers shown, we could say that:

$$\sqrt{4} = 2$$

$$\sqrt[5]{32} = 2$$

$$b^{-3} = \frac{1}{b^3}$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

In a **negative power**, the negative sign tells us to find the reciprocal of the positive power.

Step 1

Factors, multiples and primes

Step 2

Prime factorisation

Step 3

HCF and LCM

Step 4

Square and cube numbers

Step 5

Powers and roots

Step 6

Negative indices (E)

Step 7

Rational and irrational numbers, including surds

Step 8

Simplify expressions with surds (E)

Step 9

Expand brackets with surds (E)

Key vocabulary

integer	whole number that can be positive or negative
factor	integer or term that divides exactly into another integer or term
multiple	result of multiplying a number by an integer
prime number	positive integer that has exactly two factors, 1 and itself
product	result of a multiplication
highest common factor	common factor of two or more integers or terms that has the greatest value or power
lowest common multiple	lowest multiple shared by two or more numbers
prime factor	factor of a number that is prime
base	number that is multiplied when using an index
index	small number to the right and above the base number or variable that shows how many times the base is multiplied by itself
power	way to express repeated multiplication of a number using a base and an index
root	the root of a number is the value that when raised to a specific power gives the number
reciprocal	one of a pair of numbers whose product is equal to 1
surd	irrational number expressed as the root of a positive integer



Summer Block 5

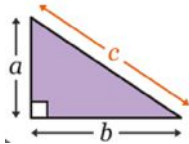
Pythagoras and trigonometry

Foundation Tier

Pythagoras' Theorem can be used to find a missing side length in a right-angle triangle when we know the other

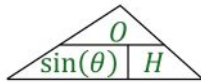
$$a^2 + b^2 = c^2 \text{ (finding the hypotenuse)}$$

$$c^2 - b^2 = a^2 \text{ (finding another side)}$$

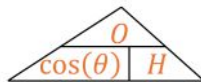


The **Sine**, **Cosine**, and **Tangent** functions can be used to calculate missing side lengths in right-angle triangles when we know one other angle and one side.

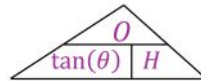
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$



$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$



$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$



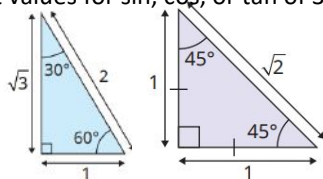
To find a missing angle in a right-angle triangle, where we know two sides, we can utilise the **inverse Sine**, **Cosine**, and **Tangent** functions.

$$\theta = \sin^{-1} \left(\frac{\text{opposite}}{\text{hypotenuse}} \right)$$

$$\theta = \cos^{-1} \left(\frac{\text{adjacent}}{\text{hypotenuse}} \right)$$

$$\theta = \tan^{-1} \left(\frac{\text{opposite}}{\text{adjacent}} \right)$$

We can utilise these triangles to help us recall the exact values for sin, cos, or tan of 30, 45, and 60.



For example:

$$\tan(60) = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\sin(45) = \frac{1}{\sqrt{2}}$$

Step 1

Pythagoras' theorem (find the hypotenuse)

Step 2

Pythagoras' theorem (find any side)

Step 3

Identify the hypotenuse, opposite and adjacent sides

Step 4

Ratios in right-angled triangles

Step 5

Use the tangent ratio to find an unknown side length

Step 6

Use the sine and cosine ratio to find an unknown side length

Step 7

Use trigonometric ratios to find an unknown side length

Step 8

Use trigonometric ratios to find an unknown angle

Step 9

Exact trigonometric values (E)

Key vocabulary

hypotenuse

longest side of a right-angled triangle that subtends the right angle

Pythagoras' theorem

rule that states that, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

opposite

side of a right-angled triangle that subtends a given angle

adjacent

side of a right-angled triangle that forms a given angle with the hypotenuse

dimensions

measurements, such as length, width and height, that describe the size and extent of an object

perpendicular

where lines, planes or surfaces meet at a right angle

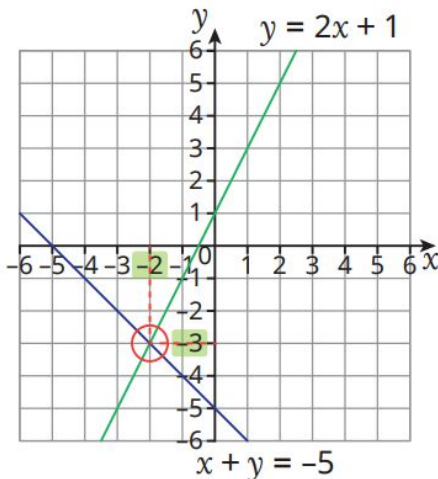


Summer Block 6

Simultaneous equations

Foundation Tier

We can solve simultaneous equations using a graph. These lines intersect at $(-2, -3)$. We can write the solutions in the form: $x = -2$
 $y = -3$



Elimination is a dependable method for solving linear simultaneous equations.

Easier:

Because our x terms match, we subtract the equations to eliminate them. Now we can solve for y .

$$\begin{array}{r} x + 4y = 17 \\ x + 2y = 9 \quad - \\ \hline 2y = 8 \quad \div 2 \\ y = 4 \end{array}$$

Once we've found one solution, substitute it into any equation to find the other. We need to write the solutions together, for our final answer.

$$\begin{array}{r} x + 2(4) = 9 \\ x + 8 = 9 \quad -8 \\ x = 1 \end{array}$$

$x = 1, y = 4$

Harder:

We need one pair of coefficients to match, so we cross multiply.

$$\begin{array}{r} 2x + 3y = 39 \quad \times 2 \\ 5x - 2y = -7 \quad \times 3 \end{array}$$

$$\begin{array}{r} 4x + 6y = 78 \\ 15x - 6y = -21 \quad + \\ \hline 19x = 57 \\ x = 3 \quad \div 19 \end{array}$$

To eliminate our matched terms, we add (different signs) or subtract (same signs).

$$\begin{array}{r} 2(3) + 3y = 39 \\ 6 + 3y = 39 \\ 3y = 33 \quad -6 \\ y = 11 \quad \div 3 \end{array}$$

$$x = 3, y = 11$$

Once we solve for the first variable, we can substitute into any equation to solve for the other. Our final answer needs a solution for each variable.

Step 1 Use one value to find another

Step 2 Introduction to simultaneous equations

Step 3 Solve simultaneous equations using graphs

Step 4 Solve simultaneous equations (no adjustments)

Step 5 Manipulating equations

Step 6 Solve simultaneous equations (adjust one)

Step 7 Solve simultaneous equations (adjust both)

Step 8 Solve simultaneous equations by substitution (E)

Key vocabulary

coefficient	number in front of a variable indicating the multiple of the variable
simultaneous equations	set of two or more equations with unknowns in common, where the solutions satisfy all equations at the same time
solve	find the value(s) or set of values of an unknown in an equation or inequality
variable	symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae
unknown	value not yet known, indicated with a letter and used in an equation or inequality
linear equation	equation that forms a straight line when graphed
coordinates	pair of numbers that shows the horizontal and vertical position of a point
intersection	point where two graphs cross
solution	value of an unknown that satisfies an equation or inequality