

YEAR 9 — REASONING WITH ALGEBRA...

Straight Line Graphs

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis

Parallel: two lines that never meet with the same gradient

Co-ordinate: a set of values that show an exact position on a graph

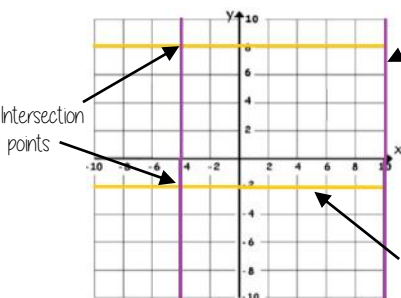
Linear: linear graphs (straight line) — linear common difference by addition/ subtraction

Asymptote: a straight line that a graph will never meet

Reciprocal: a pair of numbers that multiply together to give 1

Perpendicular: two lines that meet at a right angle

Lines parallel to the axes



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form $x = a$ and are vertical

Lines parallel to the x axis take the form $y = a$ and are horizontal

All the points on this line have a y coordinate of -2
eg (3, -2) (7, -2) (-2, -2)
all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

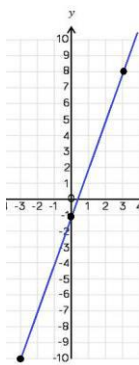
Plotting $y = mx + c$ graphs

$y = 3x - 1$ → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

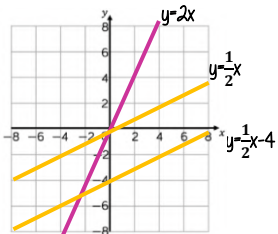
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient — the steeper the line

Positive gradients

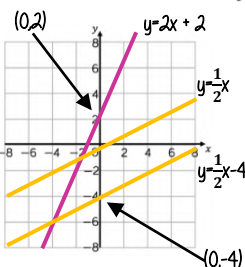
Negative gradients

Parallel lines have the same gradient

Compare Intercepts

$y = mx + c$

The value of c is the point at which the line crosses the y-axis Y intercept



The coordinate of a y intercept will always be (0, c)

Lines with the same y-intercept cross in the same place

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line

$y = mx + c$
y and x are coordinates

The value of c is the point at which the line crosses the y-axis Y intercept

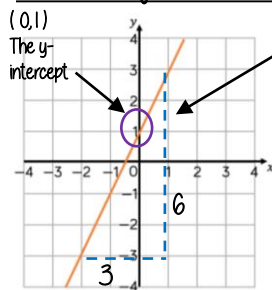
The equation of a line can be rearranged. Eg

$y = c + mx$

$c = y - mx$

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs

To represent direct proportion the graph must start at the origin

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost. The gradient shows the price per pen

The y-intercept shows the minimum charge. The gradient represents the price per mile

YEAR 9 — REASONING WITH ALGEBRA...

Forming and Solving Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

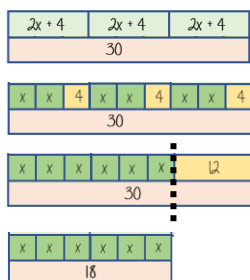
Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets

R



$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad -6$$

$$x = 3$$

$$\begin{array}{|c|} \hline x \\ \hline 3 \\ \hline \end{array}$$

Form and solve inequalities

R



Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

Inequalities with negatives

Method 1 Make x positive first

$$2 - 3x > 17$$

$$+3x \quad +3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

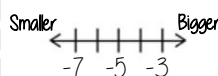
$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

✓ CHECK IT!
 $2 - 3(-6) = 20$
TRUE/ CORRECT



Equations with unknown on both sides

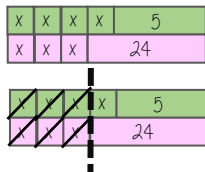
$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

Check it!

$$5(-8 + 4) < 3(-8 + 2)$$

$$5(-4) < 3(-6)$$

$$-20 < -18$$

✓ -20 IS smaller than -18

Method 2 Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

$$\downarrow$$

$$x < -5$$

When you multiply or divide x by a negative you need to reverse the inequality

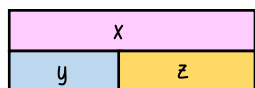
Formulae and Equations

Substitute in values

Formulae — all expressed in symbols

Equations — include numbers and can be solved

Rearranging Formulae (one step)



$$x = y + z$$

Rearrange to make y the subject

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$

Gradient = $\frac{4}{2} = 2$

YEAR 9 — REASONING WITH ALGEBRA...

Testing conjectures

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

Keywords

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors.

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Verify: the process of making sure a solution is correct

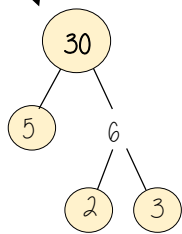
Proof: logical mathematical arguments used to show the truth of a statement

Binomial: a polynomial with two terms

Quadratic: a polynomial with four terms (often simplified to three terms)

Factors, Multiples and Primes

Multiplication part-whole models



All three prime factor trees represent the same decomposition

HCF — Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

LCM — Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share



True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4, ...
The numbers in the sequence are doubling each time.

Counterexamples



This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Always, Sometimes, Never true.

Always

Every value always supports the statement

Sometimes

Examples show the statement being true and counter examples to show when it is false.

Never

No example supports the statement

Examples to try

- 0 and 1
- Fractions
- Negative numbers

Show that

Numerical verification

Show the stages to a solution with numerical values

Algebraic verification

Show algebraic properties of the solution
You may want to use pictorial images to support this

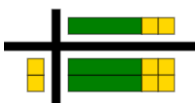
Proof

Simple proofs using algebra

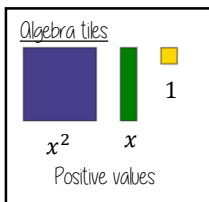
Compare the left hand side of an equation with the right hand side — are they the same or different?

Expanding binomials

$$2(x + 2) \equiv 2x + 4$$

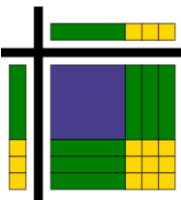


Algebra tiles can represent a binomial expansion
Has two terms



The order of the binomial has no impact on the outcome.
eg $(x + 3)(3 + x)$

$$(x + 3)(x + 3) \equiv x^2 + 6x + 9$$



This is a quadratic
It has four terms which simplified to three terms

Conjectures



Even
 $(2n)$

Multiple of 2



Odd
 $(2n + 1)$

One more than any even

Use numerical verification first
Use pictorial verification — the representations of numbers of odd and even

Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions.

Eg one space to the right of n
 $n + 1$

Eg One row below n
 $n + 10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The size of the grid for generalisation changes the relationship statements

YEAR 9 — CONSTRUCTING IN 2D/3D...

3D Shapes

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g length and width

3D: three dimensions to the shape e.g length, width and height

Vertex: a point where two or more line segments meet

Edge: a line on the boundary joining two vertex

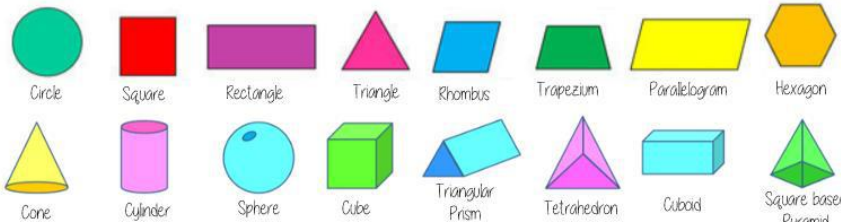
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

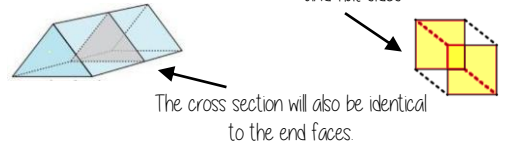
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Name 2D & 3D shapes



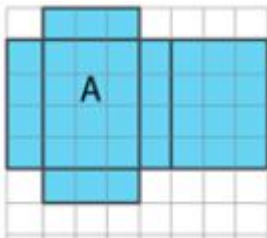
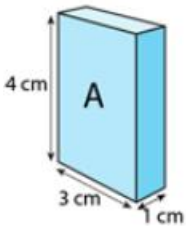
Recognise prisms

A solid object with two identical ends and flat sides



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

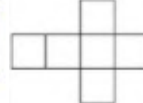
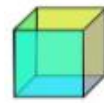
Nets of cuboids



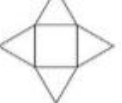
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

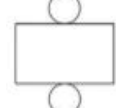
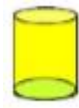
Sketch and recognise nets



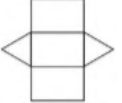
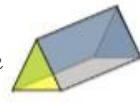
Do they have the same number of faces?



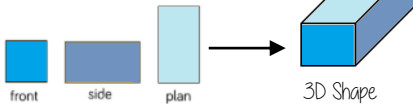
Where do the edges join?



Are the shapes of the faces correct?



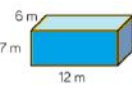
Plans and elevations



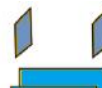
The direction you are considering the shape from determines the front and side views

Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



For cubes and cuboids you can also find one of each face and double it



Sides 6×7
 6×7
 Front and back 12×7
 12×7
 Top and Bottom 12×6
 12×6

Sum of all sides is surface area



For other shapes - not all the sides are the same, so calculate the individually

Volumes

Volume is the 3D space it takes up — also known as capacity if using liquids to fill the space



Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Prisms and cylinders = area cross section x height

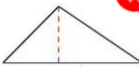
Height can also be described as depth

Area of 2D shapes

Rectangle $\text{Base} \times \text{Height}$



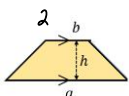
Triangle $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$



Parallelogram/ Rhombus $\text{Base} \times \text{Perpendicular height}$



Area of a trapezium $\frac{(a+b) \times h}{2}$



Area of a circle $\pi \times \text{radius}^2$



Surface area - cylinders

The area of the circle $\pi \times \text{radius}^2$



The width of this face is the same as the circumference $\pi \times \text{diameter} \times \text{height}$

$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$

Areas — square units
 Volumes — cube units

Areas and volumes can be left in terms of π

YEAR 9 — CONSTRUCTING IN 2D/3D...

Constructions & congruency

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

Protractor: piece of equipment used to measure and draw angles

Locus: set of points with a common property

Equidistant: the same distance

Discorectangle: (a stadium) — a rectangle with semi circles at either end

Perpendicular: lines that meet at 90°

Arc: part of a curve

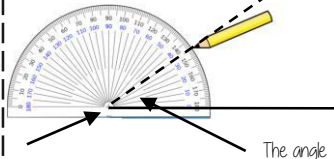
Bisector: a line that divides something into two equal parts

Congruent: the same shape and size

Draw and measure angles

Draw a 35° angle

Make a mark at 35° with a pencil
And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

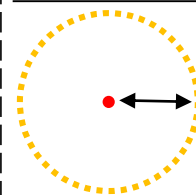
The car image is 10cm

Image : Real life
1cm : 30cm
 $\times 10$ $\rightarrow$ 10cm : 300cm $\times 10$



Locus of a distance from a point

All points are equidistant (the same distance) from the fixed point in the middle



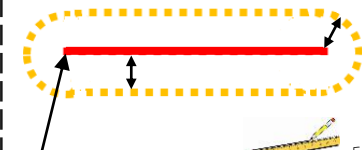
If the point is in the corner it can only make a quarter circle



Equipment needed
The radius is the distance from the fixed point

Locus of a distance from a straight line

All points are equidistant (the same distance) from line



The ends of the line are fixed points

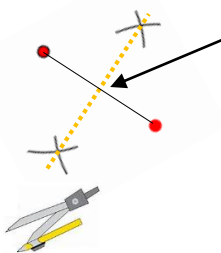


Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points

Also a perpendicular bisector

Because if the points are joined, this new line intersects it at a 90°



Join the intersections with a ruler.
All points on this line are equidistant from both points



Keep the compass the same size and draw two arcs from each point

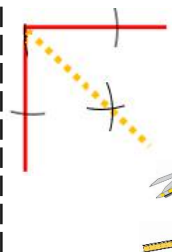
Locus of a distance from two lines

Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection



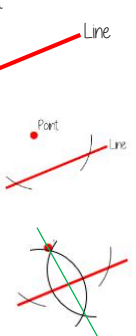
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass

Keep the compass the same distance and now use your new points to make new intersecting arcs

Connecting the arcs makes the bisector



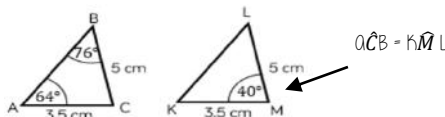
If P is a point on the line the steps are the same

Congruent figures



Congruent figures are identical in size and shape — they can be reflections or rotations of each other

Congruent shapes are identical — all corresponding sides and angles are the same size



Because all the angles are the same and $AC = KM$, $BC = LM$ triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

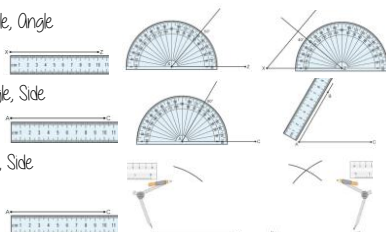
The triangles both have a right angle, the hypotenuse and one side are the same

Constructing Triangles

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



YEAR 9 — REASONING WITH NUMBER...

Numbers

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

Keywords

Integer: a whole number that is positive or negative

Rational: a number that can be made by dividing two integers

Irrational: a number that cannot be made by dividing two integers

Inverse operation: the operation that reverses the action

Quotient: the result of a division

Product: the result of a multiplication

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2:1 ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat

The square root of a negative is not a real number and cannot be found

HCF/LCM

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Standard form

Any number between 1 and less than 10 $\rightarrow A \times 10^n$ Any integer

$6 \times 10^5 + 8 \times 10^5$

= 600000 + 800000

= 1400000

= 1.4×10^6

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

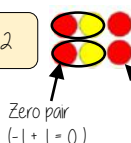
$15 \div 0.3 \times 10^5 \div 10^3$

= 5×10^2

Directed number

Addition

$$2 + -4 = -2$$



Generalisation

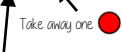
$$+ - = -$$

Subtraction

$$2 - 4 = -2$$

Representation for calculation

$$2 - -1 = 3$$



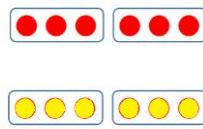
Generalisation

$$- - = +$$

"Subtract" – means take away or remove

Start with the representation of 2

Multiplication



$$-2 \times -3 = 6$$

Red = -1
Yellow = 1

The act of making counters into their negative is turning them over

Divisions are the inverse operations



$a = 5$

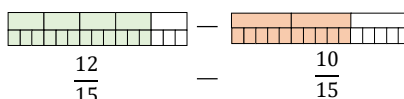
$b = -4$

Brackets around negative substitutions helps remove calculation errors

$$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$$

Addition/ Subtraction of fractions

$$\frac{4}{5} - \frac{2}{3}$$



$$= \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

Multiplication/ Division of fractions

Shade in 3 parts

Repeat it on this many rows

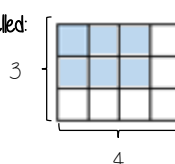
$$\frac{3}{4} \times \frac{2}{3}$$

This many columns

This many rows

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Modelled:



Parts shaded

Total number of parts in the diagram

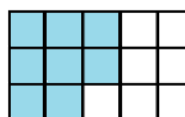
Remember to use reciprocals

$$2 \div \frac{3}{4}$$

$$2 \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

Represented



$$= \frac{8}{3}$$

YEAR 9 — REASONING WITH NUMBER...

Using Percentages

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

Percent: parts per 100 — written using the % symbol

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.

Fraction: a fraction represents how many parts of a whole value you have.

Equivalent: of equal value.

Reduce: to make smaller in value.

Growth: to increase/ to grow.

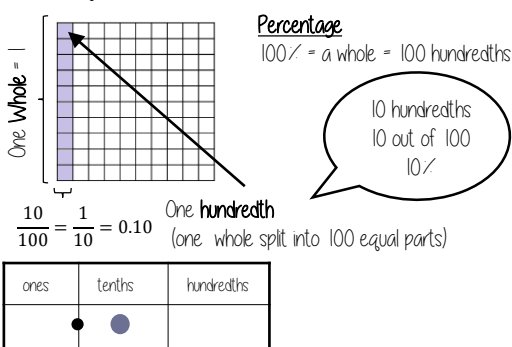
Integer: whole number, can be positive, negative or zero.

Invest: use money with the goal of it increasing in value over time (usually in a bank).

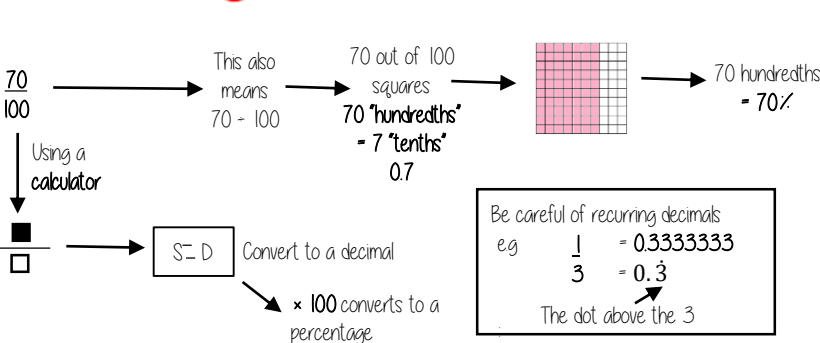
Multiplier: the number you are multiplying by.

Profit: the income take away any expenses/ costs

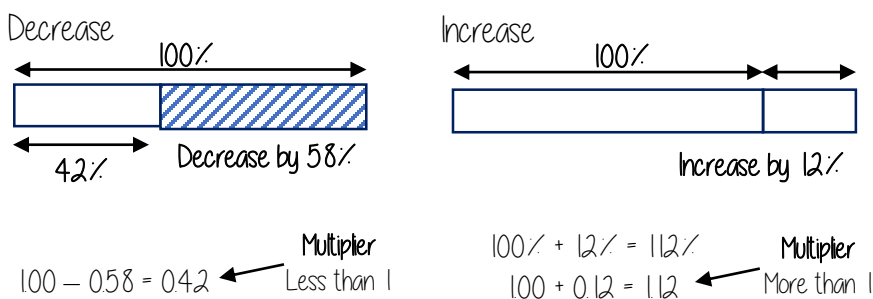
FDP Equivalence



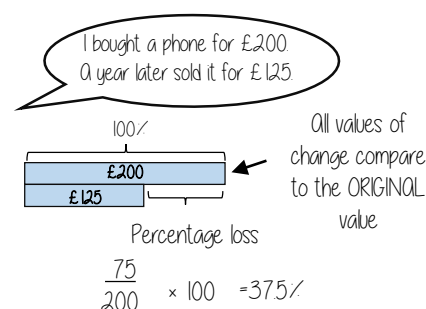
Converting FDP



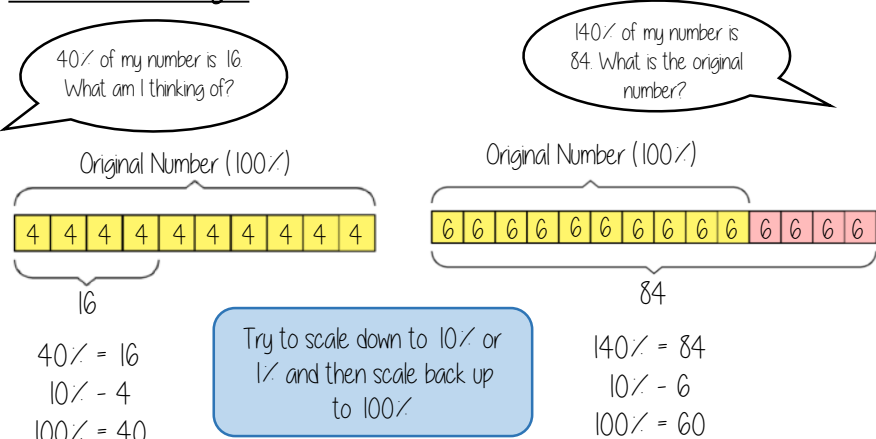
Percentage Increase/ Decrease



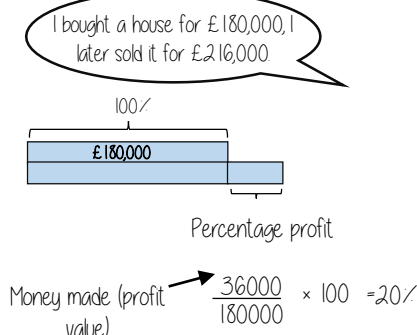
Percentage change



Reverse Percentages



$$\frac{\text{Difference in values}}{\text{Original value}} \times 100$$



YEAR 9 — REASONING WITH NUMBER...

Maths & Money

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

Keywords

Credit: money being placed into a bank account

Debit: money that leaves a bank account

Balance: the amount of money in a bank account

Expense: a cost/ outgoing

Deposit: an initial payment (often a way of securing an item you will later pay for)

Multiplier: a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)

Per Annum: each year

Currency: the type of money a country uses.

Unitary: one — the cost of one.

Bills and Bank Statements

Bills — tell you the amount items cost and can show how much money you need to pay.

Some can include a total
Look for different units
(Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Sept	Bday Money	£15		£915

Simple Interest

For each year of investment the interest remains the same.

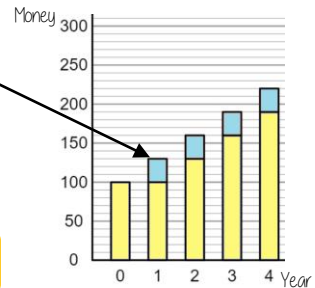
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account

e.g. Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned **£120** interest.
At the end of year 4 they have **£220**



Compound Interest

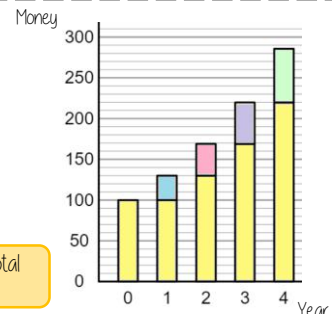
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g. Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = £285.61$$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

Wages and Taxes

Salaries fall into tax brackets — which means they pay this much each month from their salary.

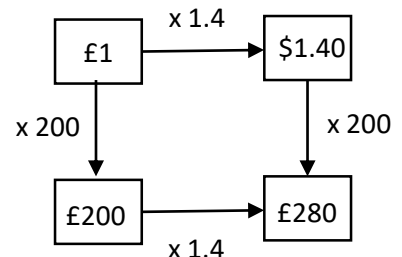
Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:

Time and a half — means 1.5 times their hourly rate

Double — 2 times their hourly rate

Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies

	£	Pounds
United Kingdom	£	Pounds
United States of America	\$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges £1	5 cupcakes £1.20
-----------------	---------------------

$$\begin{aligned} 4 &= £1.00 \div 2 & 5 &= £1.20 \div 5 \\ 2 &= £0.50 & & \\ 1 &= £0.25 \div 2 & 1 &= £0.20 \end{aligned}$$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

YEAR 9 — REASONING WITH GEOMETRY... Deduction

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify angles in parallel lines
- Solve angle problems
- Make conjectures with angles
- Make conjectures with shapes

Keywords

Parallel: two straight lines that never meet with the same gradient

Perpendicular: two straight lines that meet at 90°

Transversal: a line that crosses at least two other lines

Sum: the result of adding two or more numbers

Conjecture: a statement that might be true but is not proven

Equation: a statement that says two things are equal

Polygon: a 2D shape made from straight edges

Counterexample: an example that disproves a statement

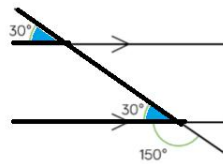
Alternate angles

Because alternate angles are equal the highlighted angles are the same size



Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°

As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

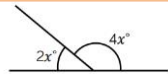


Solving angle problems

Angles on a straight Line



Link angle facts to algebra



$$2x + 4x = 180^\circ$$

The sum of angles on a straight line is 180°

$$2x + 4x = 180^\circ$$

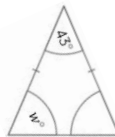
$$6x = 180^\circ$$

$$x = 30^\circ$$



Vertically opposite angles
Equal

Angles around a point
 360°

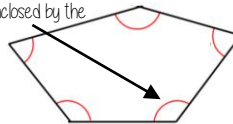


Triangles
Sum of angles is 180°

Isosceles have the same base angles

Interior Angles

The angles enclosed by the polygon



$$(\text{number of sides} - 2) \times 180$$

Making conjectures with angles



True

Always

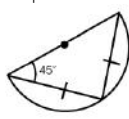
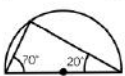
Never

False

Sometimes

Proving a conjecture

A pattern is noticed for many cases



Apply the angle rules

The sum of angles in a triangle is 180°

Test the theory

$$180 - 70 - 20 = 90$$

$$180 - 85 - 5 = 90$$

$$180 - 45 - 45 = 90$$

Make conjecture

The angle that meets the circumference in a semi circle is 90°

Disproving a conjecture

Only one counterexample is needed to disprove a conjecture

Making conjectures with shapes

Keywords and facts to recall with shape

Area: the amount of space inside a shape

Perimeter: the length around a shape

Regular Polygons: All sides and angles are equal

Quadrilateral Facts



Square

All sides equal size
All angles 90°
Opposite sides are parallel



Rectangle

All angles 90°
Opposite sides are parallel



Rhombus

All sides equal size
Opposite angles are equal



Parallelogram

Opposite sides are parallel
Opposite angles are equal
Co-interior angles



Kite

No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

YEAR 9 — REASONING WITH GEOMETRY...

Rotation & Translation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement

Symmetry: when two or more parts are identical after a transformation

Regular: a regular shape has angles and sides of equal lengths

Invariant: a point that does not move after a transformation

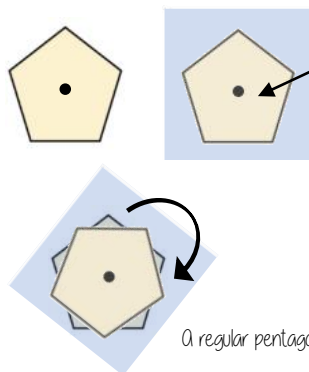
Vertex: a point two edges meet

Horizontal: from side to side

Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



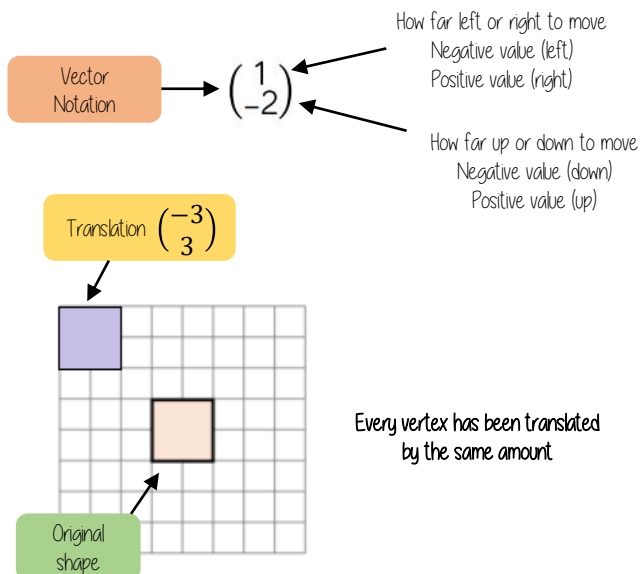
1 Trace your shape (mark the centre point)

2 Rotate your tracing paper on top of the original through 360°

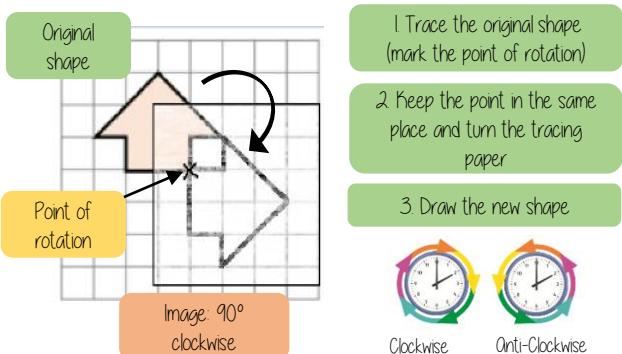
3 Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

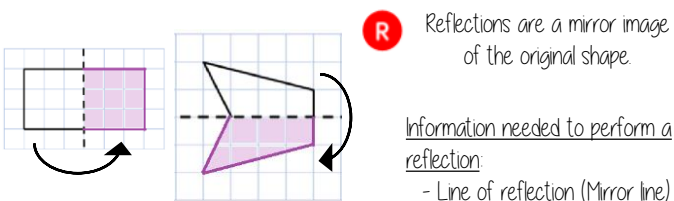
Translation and vector notation



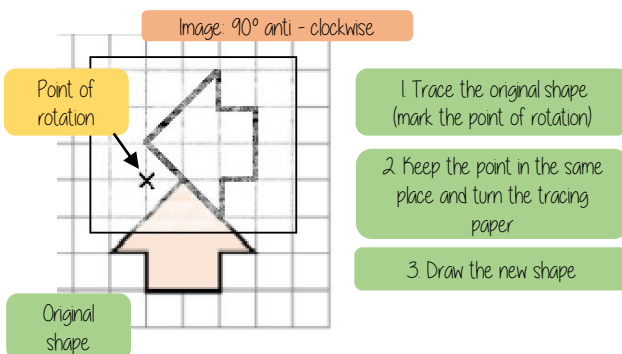
Rotate from a point (in a shape)



Compare rotations and reflections



Rotate from a point (outside a shape)



YEAR 9 — REASONING WITH GEOMETRY...

Pythagoras' theorem

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots



This can also be written as 6^2

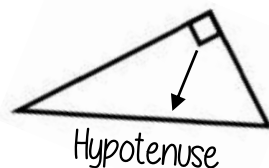
$\sqrt{\quad}$ is the square root symbol

eg $\sqrt{64} = 8$
Because $8 \times 8 = 64$

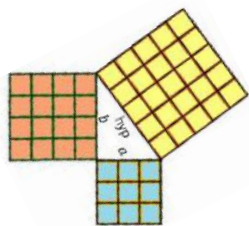
1 x 1	2 x 2	3 x 3	4 x 4	5 x 5	6 x 6	7 x 7	8 x 8	9 x 9	10 x 10
1	4	9	16	25	36	49	64	81	100

Square numbers

Identify the hypotenuse



Determine if a triangle is right-angled



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

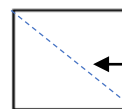
$$a^2 + b^2 = \text{hypotenuse}^2$$

eg $a^2 + b^2 = \text{hypotenuse}^2$

$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \end{aligned}$$

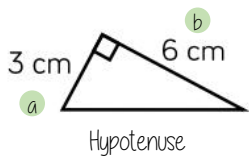
Substituting the numbers into the theorem shows that this is a right-angled triangle

The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

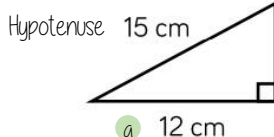
$$45 = \text{hypotenuse}^2$$

$$\sqrt{45} = \text{hypotenuse}$$

$$6.71\text{cm} = \text{hypotenuse}$$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1 Substitute in the values you are given

$$144 + b^2 = 225$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

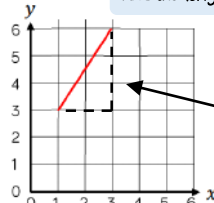
Square root to find the length of the side

$$b^2 = 111$$

$$b = \sqrt{111} = 10.54\text{ cm}$$

Pythagoras' theorem on a coordinate axis

Find the length of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram

The line segment is the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes

YEAR 9 — REASONING WITH GEOMETRY...

Enlargement & Similarity

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes

Keywords

Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

Scale Factor: the multiple describing how much a shape has been enlarged

Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)

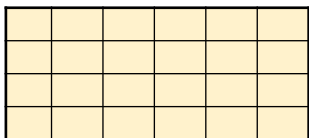
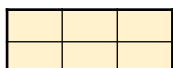
Corresponding: objects (or sides) that appear in the same place in two similar situations.

Image: the picture or visual representation of the shape

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

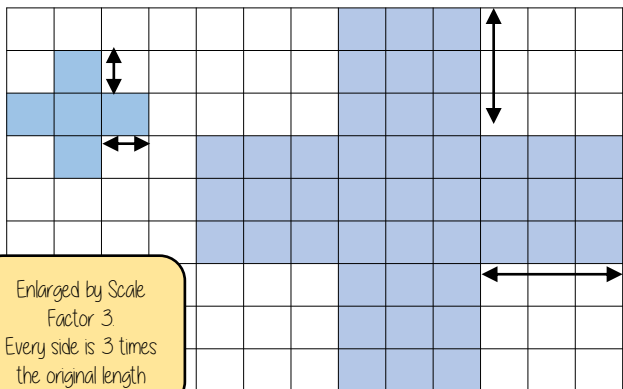
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

Enlarge by a positive scale factor

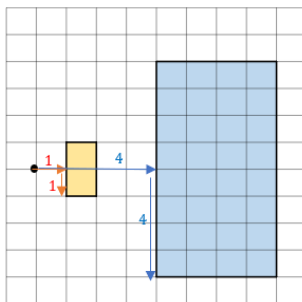
With a scale factor larger than 1 it makes the shape bigger



Enlarged by Scale Factor 3
Every side is 3 times the original length

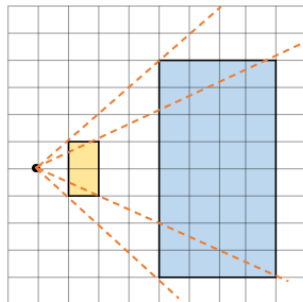
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

Rays method

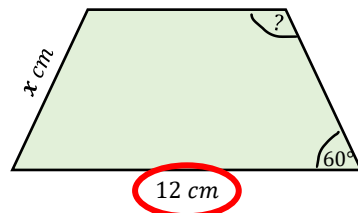
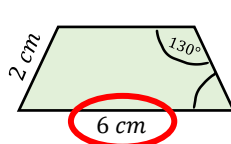


Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor

$$\frac{12}{6} = 2$$

Scale Factor = 2

Calculate the missing side

Length (corresponding side) $\times$ scale factor

$$2\text{ cm} \times 2$$

$$x = 4\text{ cm}$$

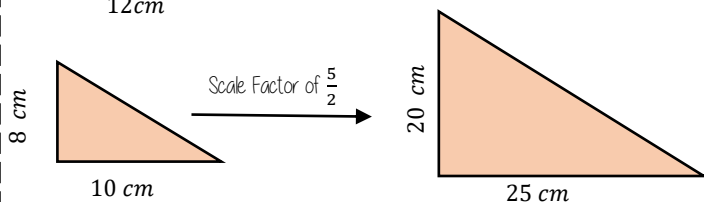
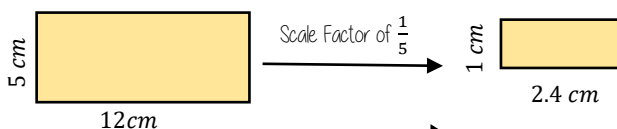
Enlargement does not change angle size

Calculate the missing angle

Corresponding angles remain the same
 130°

Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape smaller



YEAR 9 — REASONING WITH GEOMETRY...

Solving ratio & proportion problems

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

R



4 cans of pop = £2.40

$\times 0.5$ 4 cans of pop = £2.40
2 cans of pop = £1.20 $\times 50 \times$

This multiplier is the same in the same way that this would be for ratio

This is a multiplicative change

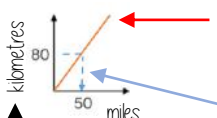
4 cans of pop = £2.40
 $\times 3$ 12 cans of pop = £7.20 $\times 3$

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables

R



Labelling of both axes is vital

This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare — then find the associated point by using your graph
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

Arrows indicate: 1 to 2 is $\times 2$, 2 to 8 is $\times 4$, 40 to 20 is $\div 2$, 20 to 5 is $\div 4$.

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts



Shop A

4 cans for £1.20
 $\downarrow$
£1.20 $\div$ 4

Cost per item

1 can is £0.30
Or 30p

Shop B

3 cans for 93p
 $\downarrow$
£0.93 $\div$ 3

1 can is £0.31
Or 31p

Shop A is the best value as it is 1p cheaper per can of pop



Shop A

4 cans for £1.20
 $\downarrow$
4 $\div$ £1.20

Cost per pound

£1 buys 3.333 cans of pop

3 cans for 93p
 $\downarrow$
3 $\div$ £0.93

£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

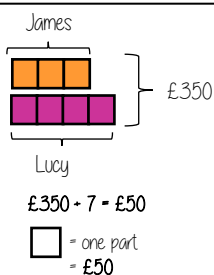
R

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



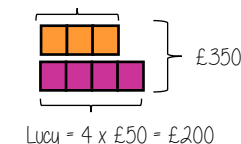
Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 $\times$ £50 = £150



Finding a value given 1:n (or n:1)

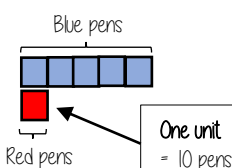
R

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

□ = one part = 10 pens

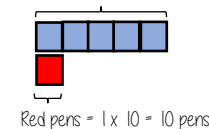


Put back into the question

Blue: Red

($\times 10$) 5 : 1 ($\times 10$)
50 : 10

Blue pens = 5 $\times$ 10 = 50 pens



There are 50 Blue Pens

YEAR 9 — REASONING WITH GEOMETRY...

Rates

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

Keywords

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

Substitute: putting numbers where letters are — replacing numbers into a formula

Speed, Distance, Time

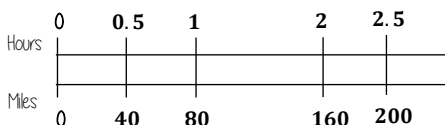
'per' for every

e.g. 80 miles per hour (mph)

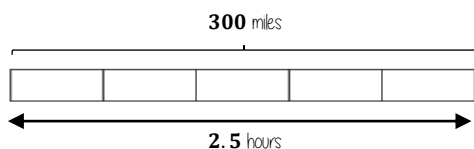
Travel 80 miles every hour

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.



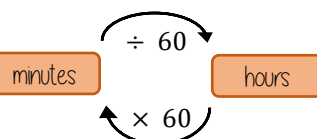
Bar models can help to calculate mph

Each part is half an hour
Each part is 60 miles



Speed, Distance, Time

Before calculations — make sure you are working in the same units as the speed



Learn or learn how to rearrange the formula for speed, distance and time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Substitute in the variables given

$$\text{distance} = \text{speed} \times \text{time}$$

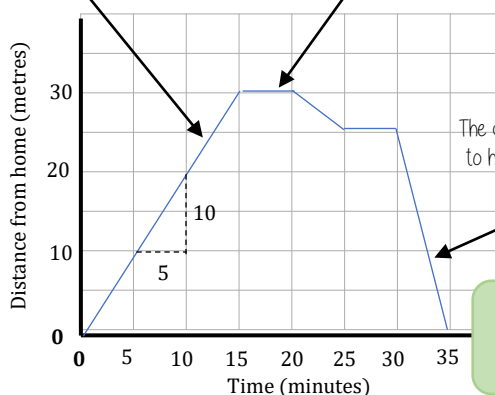
Distance — Time graphs

The steeper a gradient the faster the speed

Gradient = speed

$$\frac{10}{5} = 2 \text{ metres per min}$$

Horizontal lines represent staying still

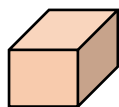


Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



$$\text{volume of prism} = \text{Area of cross section} \times \text{Depth}$$



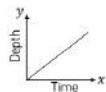
Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important
Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

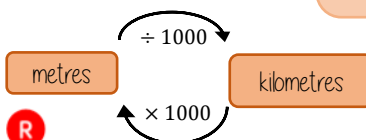
Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles per hour

Exchange rates: euros per pounds

Density: mass per volume



YEAR 9 — REPRESENTATIONS...

Probability

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

Keywords

Probability: the chance that something will happen

Relative Frequency: how often something happens divided by the outcomes

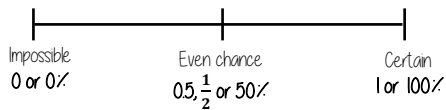
Independent: an event that is not effected by any other events.

Chance: the likelihood of a particular outcome.

Event: the outcome of a probability — a set of possible outcomes.

Biased: a built in error that makes all values wrong by a certain amount.

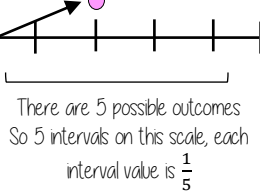
The probability scale



The more likely an event the further up the probability it will be in comparison to another event.
(It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability



There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Single event probability

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$
∴ The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections

$$\text{Relative frequency} \times \text{Number of times} \\ 0.3 \times 100 = 30$$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times

Show that dark chocolate is expected to be selected 60 times

$$0.15 \times 400 = 60$$

Independent events



The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

$$\text{Probability of event 1} \times \text{Probability of event 2}$$



$$P(5) = \frac{1}{6}$$

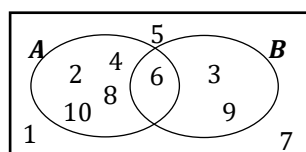
$$P(R) = \frac{1}{4}$$

Find the probability of getting a 5 and a red

$$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables



	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from tossing a coin

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

YEAR 9 — REPRESENTATIONS...

Algebraic Representation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

Keywords

Quadratic: a curved graph with the highest power being 2. Square power.

Inequality: makes a non equal comparison between two numbers

Reciprocal: a reciprocal is 1 divided by the number

Cubic: a curved graph with the highest power being 3. Cubic power.

Origin: the coordinate (0, 0)

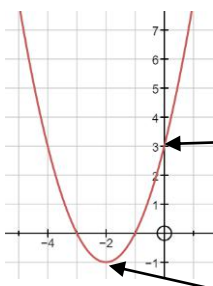
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$$y = x^2 + 4x + 3$$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

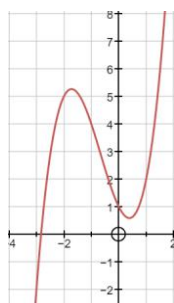
Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

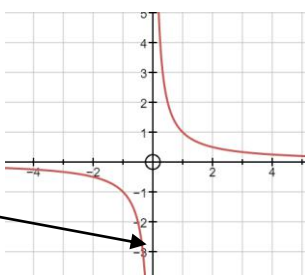
$$y = x^3 + 2x^2 - 2x + 1$$



If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

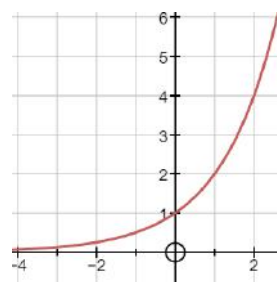
$$y = \frac{1}{x}$$



Reciprocal graphs never touch the y axis
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$$y = 2^x$$



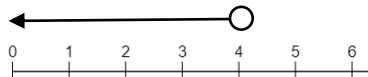
Exponential graphs have a power of x

Represent Inequalities

Multiple methods of representing inequalities

$$x < 4$$

All values are less than 4



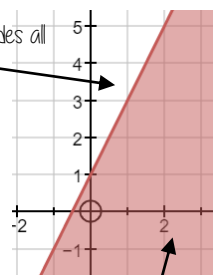
The shaded area indicates all possible values of x



The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$$y \geq 2x + 1$$



The shaded area indicates all possible solutions to this inequality