YEAR 11 - UNIT 1
GRADIENTS & LINES

Sparx Codes: U315, U377, U477, U669, U741, U848, U898

What do I need to be able to do?

- Find the equation of a straight line (from a graph, from one point & the gradient, and from two points)
- Decide whether a point is on a line
- Recognise & find equations for perpendicular lines
- Solve simultaneous equations graphically

Keywords

Parallel - parallel lines have the same gradient, so they never meet

Perpendicular - perpendicular lines meet at 90°

Gradient - how steep a line is (change in y ÷ change in x)

y-intercept - where a line crosses the y-axis

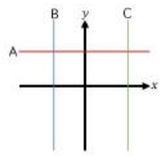
Simultaneous equations - equations where the variables have the same values

Reciprocal - one over (divided by) a given number

Lines parallel to the axis (R)

A is parallel to the **x-axis**.

B and C are parallel to the **y-axis**.



$y = 3$ is parallel to the **x-axis**.

$x = -2$ is parallel to the **y-axis**.

Plot straight line graphs (R)

Complete the table for $y = 3x + 2$.

x	−2	−1	0	1	2
y	-4	-1	2	5	8

3 X −2 + 2 = -6 + 2 = **-4**

3 X −1 + 2 = -3 + 2 = **-1**

3 X 0 + 2 = 0 + 2 = **2**

3 X 1 + 2 = 3 + 2 = **5**

Interpret y = mx + c (R)

$y = mx + c$

m = gradient (change in y ÷ change in x)

c = y-intercept (where the line crosses the y axis)

Parallel lines have the same gradient (m)

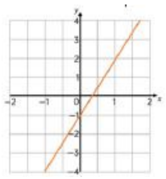
$y = 3x$ $\quad y = 3x + 1$

$y = 3x + 2$ $\quad y = 3x + 5$

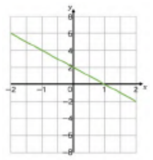
Equation of a line from a graph (R)

A graph with a positive gradient moves upwards as it moves right.

A graph with a negative gradient moves downwards as it moves right.



positive



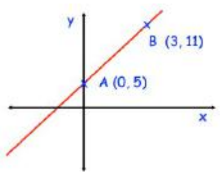
negative

Equation of a straight line from points

Gradient = $\frac{dy}{dx} = \frac{11 - 5}{3 - 0} = \frac{6}{3} = 2$

Y-intercept = 5 (from the graph)

So, our equation is $y = 2x + 5$



Determine if a point is on a line

We can use substitution to see whether a point is on a given line.

Is (2, 7) on the line $y = 3x + 1$?

$x = 2 \quad y = 7$

$y = 3x + 1$

$7 = 3 \times 2 + 1$

$7 = 6 + 1$

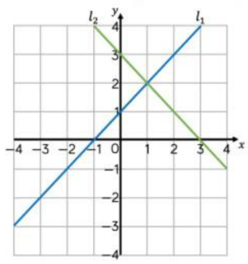
$7 = 7$ —> This is true, so (2, 7) is on the line!

Solving simultaneous equations graphically (R)

The lines l_1 and l_2 are shown on a graph.

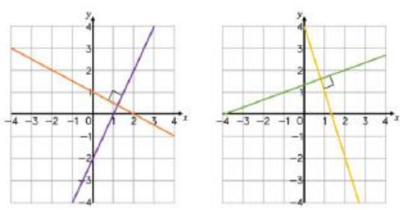
They cross at the point (1, 2).

Therefore the equations for lines l_1 and l_2 have a solution of $x = 1$, $y = 2$.



Recognising perpendicular lines (H)

Each of the graphs below shows two **perpendicular** lines. They are **perpendicular** because the meet at a 90° angle.



Equations of perpendicular lines (H)

If two lines are perpendicular, their gradients are the **negative reciprocal** of the other.

L1 and L2 are perpendicular.

L1: $y = 3x$

L2: $y = -\frac{1}{3}x$

Since they are perpendicular, their gradients should multiply to -1.

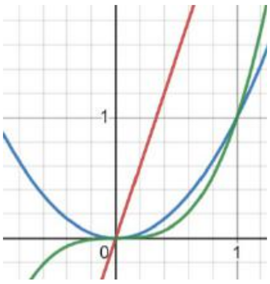
$-\frac{1}{3} \times 3 = -1$

YEAR 11 - UNIT 2
NON-LINEAR GRAPHS

Sparx Codes: U667, U989, U980, U593, U601, U229, U567

What do I need to be able to do?

- Recognise, sketch, and interpret graphs of linear, quadratic, cubic, reciprocal, and exponential functions
- Plot graphs using a table of values
- Find approximate solutions using a graph
- Identify and interpret roots and intercepts of quadratic functions graphically
- Recognise and use the equation of a circle with centre (0, 0)
- Draw a tangent to a given curve
- Find the gradient of a given tangent line



linear
$y = 3x$
quadratic
$y = x^2$
cubic
$y = x^3$

Keywords

Quadratic graph - a graph involving x^2 (U or n-shaped curve)

Cubic graph - a graph involving x^3

Reciprocal graph - a graph involving x in the denominator (e.g. $y = 1/x$ or $y = 5/x$)

Asymptote - a line that is approached, but never met, by a curve

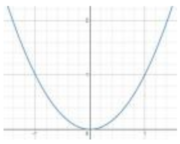
Exponential graph - a graph involving a power of x, which increase rapidly (e.g. $y = 3^x$ or $y = -2^x$)

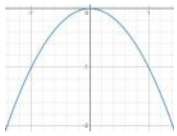
Pythagoras' theorem - a rule used to work out missing lengths in right-angle triangles ($a^2 + b^2 = c^2$)

Tangent - a straight line that meets a curve at exactly one point (but doesn't cross it)

Plot and read from quadratic graphs

A quadratic graph with a **positive** x^2 coefficient has a **U-shape** (e.g. $y = x^2$ or $y = 3x^2$)

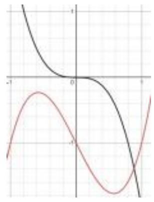




A quadratic graph with a **negative** x^2 coefficient has an **n-shape** (e.g. $y = -x^2$ or $y = -3x^2$)

Plot and read from cubic graphs

Cubic graphs can vary in appearance. Some have two turning points, some have none. All cubic graphs tend towards $\pm\infty$ on the y-axis.



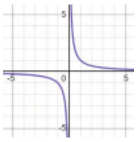
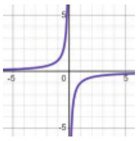
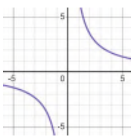
Just like other equations, we can use substitution to produce a table of values for cubic graphs.

$y = x^3$

x	−3	−2	−1	0	1	2	3
y		−8					27

Plot and read from reciprocal graphs

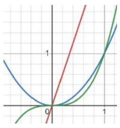
Reciprocal equations involve dividing by x. They never cross the x-axis or y-axis. This means the axes are **asymptotes** - lines that are approached, but never crossed.

$y = 1/x$ $y = -1/x$ $y = 7/x$

Recognise graph shapes

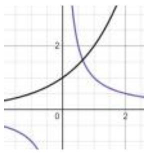
We can identify the type of graph by looking for key features.



linear - straight lines
quadratic - U or n-shape curves
cubic - 0 or 2 turning points, tend to $\pm\infty$

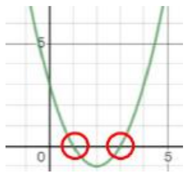
reciprocal - axes are both asymptotes (don't cross them)

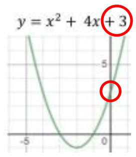
exponential - rapidly gets steeper (increasing gradient)



Identify and interpret roots and intercepts of quadratics

A **root** is where the graph crossed the x-axis. Quadratics can cross the x-axis 0, 1, or 2 times. This graph has roots $x = 1$ and $x = 3$.

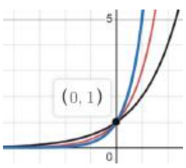




The y-intercept is where the graph crosses the y-axis. This graph has a y-intercept of (0, 3).

Understand and use exponential graphs (H)

Exponential graphs form a curve that increases rapidly. Their gradient increases as we move along the x-axis. They are always entirely above the x-axis.

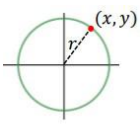


Exponential graphs typically involve an equation of the form $y = a^x$. These graphs always have a y-intercept of (0, 1).

Find and use the equation of a circle with centre (0,0) (H)

$x^2 + y^2 = r^2$

Circles with centre (0, 0) have equations of the form above, where r is the circle's radius.



$x^2 + y^2 = 25$ -> centre of (0,0) and radius of 5

$x^2 + y^2 - 36 = 0$ -> centre of (0,0) and radius of 6

$x^2 = -y^2 + 81$ -> centre of (0,0) and radius of 9

$x^2 + y^2 = 18$ -> centre of (0,0) and radius of $3\sqrt{2}$

Find the equation of the tangent to any curve (H)

A **tangent** to a curve meets the curve at exactly one point - it doesn't cross through it. A **tangent** line can be used to find the gradient of a curve at a given point.

This is **not** a tangent line as it crosses the curve twice.

This is a tangent line as it only meets the curve once.

YEAR 11 - UNIT 3
USING GRAPHS

Sparx Codes: U638, U462, U403, U914, U562, U937, U611, U238, U601, U836, U882

What do I need to be able to do?

- Plot and interpret graphs of non-standard functions in real contexts e.g. kinematic problems involving distance, speed, and acceleration
- Apply the concepts of instantaneous and average rate of change
- Calculate and interpret gradients and areas under graphs

Keywords

Direct proportion - as one quantity increases, the other quantity increases (proportionally)

Inverse proportion - as one quantity increases, the other quantity decreases (proportionally)

Gradient - how steep a line is (change in y ÷ change in x)

Distance - the length of a path travelled (typically measured in metres, kilometres, or miles)

Speed - the rate at which an object travels distance (typically measured in m/s or mph)

Acceleration - the rate at which an object changes speed (typically measures in m/s^2)

Trapezium - a quadrilateral with *at least* one pair of parallel sides

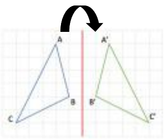
Reflect shapes in given lines R

Horizontal lines: $y = 3$ $y = -2$ $y = 0.5$

Vertical lines: $x = 6$ $x = -1$ $x = -2.3$

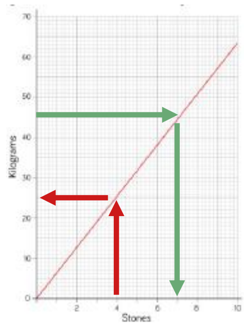
Diagonal lines: $y = 3x - 1$ $y = -2x + 7$

When a shape is reflected, the new shape will always be the **same distance** from the mirror line as the original.



Construct and interpret conversion graphs R

We can use a conversion graph to easily convert from one unit into another.



45kg ≈ 7 stone

4 stone ≈ 25kg

Construct and interpret other real-world straight line graphs R

Graphs can be used to represent a wide range of real-world scenarios.

A journey of 10 miles would cost around £15.50.

The minimum cost of a journey is £8.

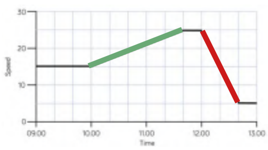
Construct and Interpret distance/time graphs

- A **horizontal** line represents **no movement**
- The **gradient** represents **speed** (straight lines indicate a constant speed)
- A positive gradient indicates movement away from the starting point
- A negative gradient indicates movement towards the starting point

Construct and interpret speed/time graphs

Speed/time graphs look very similar to distance/time graphs - read the axis labels carefully!

- A **horizontal** line represents a **constant speed**
- The **gradient** represents **acceleration**

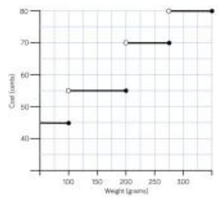


Accelerating

Decelerating

Construct and interpret piece-wise graphs

Piece-wise graphs have different rules for different values. This graph shows the cost to post a letter based on its weight.

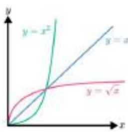


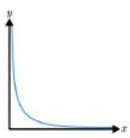
● indicates the value is included

○ indicates the value is **not** included

Recognise and interpret graphs that illustrate direct and inverse proportion

Direct proportion is represented by graphs that pass through the origin (0, 0). They are always of the form $y = kx^n$

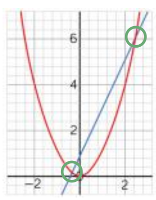




Inverse proportion is represented by a reciprocal graph in the first quadrant. They never touch either axis.

Find approximate solutions to equations using graphs

When looking for the solutions to a pair of simultaneous equations, we can look for where the lines cross over.



These graphs cross in two locations, so there are two pairs of solutions.

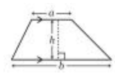
$x \approx -0.3$ $y \approx 0.2$

$x \approx 2.4$ $y \approx 6$

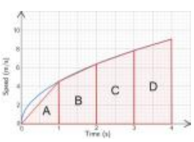
Estimate the area under the curve H

We can estimate the area under a curve by splitting the area into trapeziums and adding the areas.

$\boxed{A = (a + b) \times h \div 2}$



The curve below has been split into 4 sections. We'll work out the first 2 - try the rest yourself.



$Area_A = (0 + 4.5) \times 1 \div 2 = 2.25$

$Area_B = (4.5 + 6.4) \times 1 \div 2 = 5.45$

YEAR 11 - UNIT 4
EXPANDING & FACTORISING

Sparx Codes: U179, U768, U606, U365, U178, U228, U960, U963, U665

What do I need to be able to do?

- Recognise equations and identities
- Use a variety of methods to factorise quadratic expressions and solve quadratic equations
- Use algebra to construct arguments and proofs
- Solve simultaneous equations graphically

Keywords

Factorise - taking the highest common factor of all terms outside of a bracket

Expand - multiply a bracket by the term outside the bracket

Expression - a combination of numbers and variables with **no** equals sign (or inequality symbol)

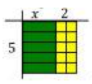
Equation - a combination of numbers and variables with an equals sign

Identity - an equation that is true for all possible values of each variable

Quadratic - involving x^2 (and no higher powers of x)

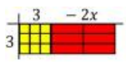
Expand and factorise with a single bracket (R)

We can expand brackets by multiplying **each term** inside the bracket by the term outside.



$5(x+2) \equiv 5x + 10$

We can factorise by dividing by the highest common factor of the terms in the brackets.



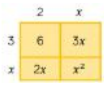
$9 - 6x \equiv 3(3 - 2x)$

Expand binomials (R)

There are a variety of models we can use to expand multiple brackets.

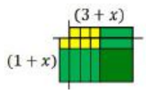
$(2 + x)(3 + x) \equiv x^2 + 5x + 6$

Multiplication grid



$(3 + x)(1 + x) \equiv x^2 + 4x + 3$

Algebra tiles



$(x + 5)(x - 3) \equiv x^2 + 2x - 15$

FOIL

First: $x \times x = x^2$

Outer: $x \times -3 = -3x$

Inner: $5 \times x = 5x$

Last: $5 \times -3 = -15$

$2x$

Factorise quadratic expressions

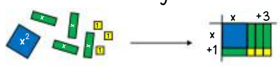
When factorising a simple quadratic expression, we look for a pair of numbers that **add to the x coefficient**, and **multiply to the number term**.

$x^2 + 9x + 14$

7 and 2 add to 9   7 and 2 multiply to 14

$x^2 + 9x + 14 \equiv (x + 7)(x + 2)$

We can also use algebra tiles to factorise.



Factorise complex quadratic expressions (H)

If our quadratic expression has an x^2 coefficient other than 1, we multiply our number term by the x^2 coefficient.

$\underline{3} \times \underline{10} = 30$

$\underline{3}x^2 + 17x \underline{+ 10}$

15 and 2 add to 17   15 and 2 multiply to 30

split the x term:

$3x^2 + 15x + 2x + 10$

factorise 2 parts: rearrange:

$3x(x + 5) + 2(x + 5) \;\rightarrow\; (3x + 2)(x + 5)$

Solve equations equal to 0

To solve an equation, we rearrange to get the variable by itself e.g. $x = 15$. We use inverse **operations** to achieve this.

Dexter and Amir are solving $1 - 3x = 0$

Dexter's method

$1 - 1 = 0$

So $3x = 1$

$x = \frac{1}{3}$

Amir's method

$1 - 3x = 0$

$1 = 3x$ + 3x to both sides

$\frac{1}{3} = x$ ÷ both sides by 3

Solve quadratic equations by factorisation

Once a quadratic equation has been factorised, we solve by making each bracket equal 0.

$x^2 + 9x + 14 = 0$	$x^2 + 7x - 18 = 0$
$(x + 7)(x + 2) = 0$	$(x + 9)(x - 2) = 0$
$x = -7$ or $x = -2$	$x = -9$ or $x = 2$

The solutions are also the negative of the bracketed number.

Solve complex quadratic equations by factorisation (H)

Once a quadratic equation has been factorised, we solve by making each bracket equal 0.

$3x^2 + 17x + 10 = 0$	$15x^2 - x - 2 = 0$
$(3x + 2)(x + 5) = 0$	$(5x - 2)(3x + 1) = 0$
$x = -2/3$ or $x = -5$	$x = 2/5$ or $x = -1/3$

The solutions are also the negative of the bracketed number divided by the x coefficient.

Complete the square (H)

Solve by completing the square:

$x^2 + 10x + 1 = 0$

$(x + 5)(x + 5) - 25 + 1 = 0$

Half the x coefficient for the brackets, subtract that value squared outside the brackets.

$(x + 5)(x + 5) - 24 = 0$

$+24$

$(x + 5)^2 = 24$

Change the two brackets to one bracket squared.

$\sqrt{\ }$

$(x + 5) = \pm\sqrt{24}$

-5

$x = -5 \pm\sqrt{24}$

Note: $(x + 5)(x + 5) - 24 = 0$ is completed square form.

Solve quadratic equations using the quadratic formula (H)

We can use the quadratic formula to solve any quadratic equation.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$3x^2 + 5x - 7 = 0$

$a = 3, b = 5, c = -7$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-7)}}{2(3)}$

$= \frac{-5 \pm \sqrt{25 + 84}}{6}$

$= \frac{-5 \pm \sqrt{109}}{6}$

$= \frac{-5 + \sqrt{109}}{6}$ or $\frac{-5 - \sqrt{109}}{6}$

$= 0.907$ or -2.573

YEAR 11 - UNIT 5
CHANGING THE SUBJECT

Sparx Codes: U237, U759, U556, U413, U573, U434, U779, U168

What do I need to be able to do?

- Recognise equations and identities
- Solve linear equations and inequalities
- Use algebra to construct arguments and proofs
- Form, solve, and interpret equations from real-world situations
- Find approximate solutions to equations using iteration
- Use inverse operations to change the subject of a formula

The **subject is bold** in each formula

$\mathbf{F} = ma$

$a^2 + b^2 = \mathbf{c^2}$

$\mathbf{V} = \pi r^2 h$

Keywords

Solve - rearrange an equation or inequality to get the variable by itself (find its value)

Equation - a combination of numbers and variables containing an equals sign

Inequality - a combination of numbers and variables containing an inequality sign (<, >, ≤, or ≥)

Formula - a rule that uses mathematical symbols to show how quantities are related

Subject - the variable from a formula that is being worked out (generally by itself in the formula)

Rearrange - using inverse operations to change the way an equation, inequality, or formula looks

Iteration - a repeated calculation, where the output is used in the next calculation

Solve linear equations (R)

To solve an equation, we rearrange to get the variable by itself e.g. x = 15. We use inverse **operations** to achieve this.

$2x + 3 = 13$ $\quad -3$

$2x = 10$ $\quad \div 2$

$x = 5$

$x/4 - 1 = 2$ $\quad +1$

$x/4 = 3$ $\quad \times 4$

$x = 12$

Solve inequalities (R)

We can solve inequalities using the same methods we use for equations. The only exception is that multiplying/dividing by a negative number changes the inequality sign.

$4x + 3 > 7$ $\quad -3$

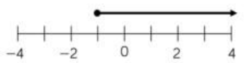
$4x > 4$ $\quad \div 4$

$x > 1$

We can also represent the solution to an inequality on a number line.

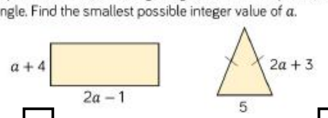
● indicates the value is included

○ indicates the value is not included



Form and solve equations and inequalities in the context of shape

The perimeter of the rectangle is greater than the perimeter of the triangle. Find the smallest possible integer value of a.



A

$2(a + 4) + 2(2a - 1) > 2(2a + 3) + 5$

$2a + 8 + 4a - 2 > 4a + 6 + 5$

$6a + 6 > 4a + 11$

B

$6a + 6 > 4a + 11$

$2a > 5$

$a > 2.5$

So the smallest possible integer value of a is 3!

Change the subject of a simple formula (R)

We can rearrange a formula using inverse operations, just like solving an equation. The goal is to get the desired subject by itself in the formula.

Make a the subject:

$F = ma$ $\quad \div m$

$F/m = a$

$a = F/m$

Make c the subject:

$y = mx + c$ $\quad -mx$

$y - mx = c$

$c = y - mx$

Change the subject of a known formula

Make r the subject of the formula:

$A = \pi r^2$ $\quad \div \pi$

$A \div \pi = r^2$ $\quad \sqrt{\ }$

$\sqrt{A \div \pi} = r$

$r = \sqrt{A \div \pi}$

Change the subject of a complex formula

Make x the subject of the formula:

$A = \left(\frac{\sqrt{x} - b}{3}\right)^2$ $\quad \sqrt{\ }$

$\sqrt{A} = \frac{\sqrt{x} - b}{3}$ $\quad \times 3$

$3\sqrt{A} = \sqrt{x} - b$ $\quad +b$

$3\sqrt{A} + b = \sqrt{x}$ $\quad ^2$

$(3\sqrt{A} + b)^2 = x$

$x = (3\sqrt{A} + b)^2$

Change the subject where the subject appears more than once (H)

If the desired subject appears more than once, we aim to get all such terms on one side of the equation, then factorise.

Make t the subject of the formula.

$5t + a = xb + xt$ $\quad -xt \ -a$

$5t - xt = xb - a$

$t(5 - x) = xb - a$ $\quad \div(5 - x)$

$t = \frac{xb - a}{5 - x}$

Solve equations by iteration (H)

An iterative formula is used repeatedly to find an approximate solution.

x_0 or u_0 tends to mean the initial or "0th" term.

x_1 or u_1 tends to mean the first term.

x_n or u_n tends to mean the current term.

x_{n+1} or u_{n+1} tends to mean the next term.

YEAR 11 - UNIT 6

FUNCTIONS

Sparx Codes: M428, U637, U448, U895, U996, U989, U133, U450

What do I need to be able to do?

- Interpret expressions as functions
- Interpret the reverse process as the inverse function and the succession of two functions as the composite function
- Solve simultaneous equations graphically
- Identify and interpret roots and turning points
- Solve and represent linear and quadratic inequalities
- Recognise, sketch, and interpret graphs of quadratic functions

f(x) -> a function of x

g(x) -> a different function of x

Keywords

Function - a relationship between an input and an output

Inverse operation - the opposite operation (should cancel each other out)

Composite function - a function made out of other functions

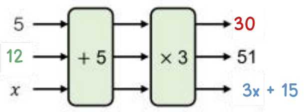
Inverse function -the opposite function (undoes the action of the original function)

Root - where a function equals zero

Turning point - a point at which a curve changes direction e.g. from having a negative gradient to having a positive gradient

Use function machines (R)

We can substitute an input or an output into a function machine. If we substitute an output, we need to use the **inverse operations** in the **opposite order**.



$5 + 5 = 10$ $10 \times 3 = 30$ $x + 5 = x + 5$

$51 \div 3 = 17$ $17 - 5 = 12$ $3 \times (x + 5) = 3x + 15$

Substitution into expressions and formulae (R)

Substitution involves replacing a variable (letter) with a given value.

Substitute $a = 3$ into each expression below.

$5a = 5 \times 3 = 15$

$(a - 5)^2 = (3 - 5)^2 = (-2)^2 = 4$

$a^2 - 2a + 4 = 3^2 - 2 \times 3 + 4 = 9 - 6 + 4 = 7$

Use function notation

Given that $f(x) = 3x + 1$:

calculate $f(2)$: $3 \times 2 + 1 = 6 + 1 = 7$

calculate $f(-0.5)$: $3 \times -0.5 + 1 = -1.5 + 1 = -0.5$

solve $f(x) = 16$:

$3x + 1 = 16$ $\quad -1$

$3x = 15$ $\quad \div 3$

$x = 5$

Work with composite functions (H)

With composite functions, always perform the inner calculation first.

$f(x) = 2x$ $\qquad g(x) = 5x - 1$

Calculate $fg(4)$: $\qquad g(4) = 5 \times 4 - 1 = 19$

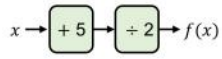
$f(g(4)) = f(19) = 2 \times 19 = \mathbf{38}$

Write an expression for $gf(x)$:

$g(f(x)) = g(2x) = 5(2x) - 1 = 5 \times 2x - 1 = 10x - 1$

Work with inverse functions (H)

Inverse functions are performed using inverse operations in the opposite order.



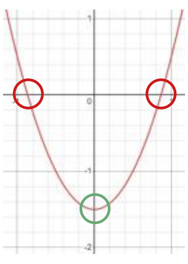
$f(x) = \frac{x + 5}{2}$ $\qquad f^{-1}(x) = 2x - 5$

For complex functions, this is more challenging.

1) Replace f(x) with y in your function.
2) Solve for x.
3) Change x to $f^{-1}(x)$, change each y to an x.

Graphs of quadratic functions

Quadratic graphs always have a U shape (positive x^2 coefficient) or n shape (negative x^2 coefficient).

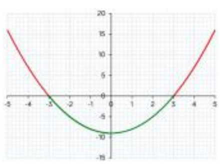


This quadratic function has roots of $x \approx -0.9$ and $x \approx 0.9$.

This quadratic function has a turning point at (0, -1.5).

Solve quadratic inequalities (H)

This graph shows the function $f(x) = x^2 - 9$.



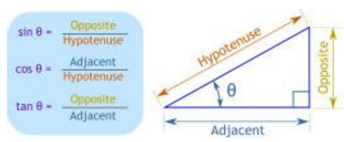
Show where $f(x) > 0$.

Show where $f(x) < 0$.

Solve $x^2 - 9 \leq 0$: $\quad -3 \leq x \leq 3$

Understand and use trigonometric functions (R)

Sine, cosine, and tangent are trigonometric ratios. We can use them to **find a missing side**, if we **know one side and one angle**. We can use them to find a **missing angle**, if we **know two sides**.



YEAR 11 - UNIT 7
MULTIPLICATIVE REASONING

Sparx Codes: U519, U134, U721, U407, U640, U842, U357, U138, U364, M478

What do I need to be able to do?

- Compare lengths, areas, and volumes using ratio notation and scale factors
- Understand the links between inverse proportion and reciprocals
- Construct and interpret equations that describe direct and inverse proportion
- Work with proportional relations algebraically and graphically

Keywords

Enlargement - a transformation that proportionally changes the size of a shape

Scale factor - how much each length in a shape is multiplied by during an enlargement

Congruent - two shapes are congruent if they have exactly the same lengths and angles (the shape can be rotated, reflected, or translated)

Similar - two shapes are similar if they have the same proportions i.e. they are an enlargement of one another (note: all congruent shapes are also similar)

Direct proportion - as one quantity increases, the other quantity increases (proportionally)

Inverse proportion - as one quantity increases, the other quantity decreases (proportionally)

Constant of proportionality - the constant value of the ratio between two proportional quantities

Pressure - the ratio of force to the area where the force is applied (often measured in N/m^2)

Density - the ratio of the mass of an object to its volume (often measured in kg/m^3, g/cm^3, or similar)

Use scale factors (R)

When we enlarge a shape, all sides are multiplied by the same value (a scale factor). All angles stay the same.

4 ÷ 6 = 0.666...

15 X 0.666... = 10

x = 10cm

Understand direct proportion

In direct proportion, both quantities increases or decrease proportionally with each other. If one value doubles, so does the other. If one quantity halves in value, so does the other.

If one quantity is zero, so is the other. So direct proportion graphs always pass through the origin (0, 0).

Construct complex direct proportion equations (H)

Direct proportion can be written as an equation of the form $y = kx$ (we may use different letters).

x and y are variables, so they can change.

k is called the constant of proportionality - it is the same for all values of the variables.

$p = kt$ when $p = 100$, $t = 5$. Find k.

$100 = k \times 5$ $\quad 100 \div 5 = k$ $\quad k = 20$ -> $p = 20t$

Calculate with pressure and density

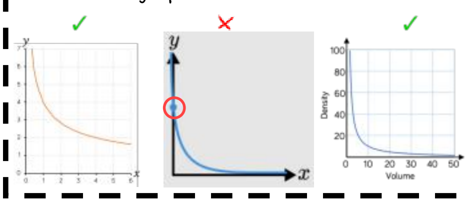
$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$ $\qquad$ $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

Pressure and density are compound measures, as we use two quantities to work them out. If we know any two values from one of these equations, we can work out the missing third value.

Understand inverse proportion

With inverse proportion, one quantity increases as the other decreases proportionally (like a reciprocal graph).

With inverse proportion, neither value can reach zero, so their graphs never touch either axis.



Construct inverse proportion equations (H)

Inverse proportion can be written as an equation of the form $y = \frac{k}{x}$ (may use different letters).

x and y are variables, so they can change.

k is called the constant of proportionality - it is the same for all values of the variables.

$a = \frac{k}{e}$ when $a = 13$, $e = 2$. Find k.

$13 = k \div 2$ $\quad 13 \times 2 = k$ $\quad k = 26$ -> $a = \frac{26}{e}$

Ratio problems (R)

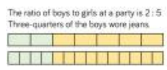
A ratio is a way of splitting something into parts. Here are some ways to make solving ratio problems more manageable:

1 - Label the parts of the ratio using letters

a	b	Total
3 :	5	8

2 - Don't add/subtract to a ratio - only multiply/divide

3 - Consider how you can represent the problem visually



YEAR 11 - UNIT 8
GEOMETRIC REASONING

Sparx Codes: U390, U826, U121, U427, U471, U781, U459, U251, U130, U489, U951, U808, U636

What do I need to be able to do?

- Use deductive reasoning to solve geometry problems
- Apply circle theorems
- Interpret and use bearings
- Perform addition, subtraction, and scalar multiplication to column vectors
- Use vectors to construct proofs

Keywords

Vertically opposite angles - angles opposite each other in an X-shape (always equal)

Parallel - parallel lines have the same gradient, so they never meet

Radius - a straight line from the centre of the circle to the circumference (edge)

Chord - a line segment connecting two points on a curve e.g. two points on a circle's circumference

Segment - the smaller region created when a circle is split by a chord

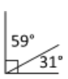
Sector - a slice of a circle, formed by two radiuses and part of the circumference

Tangent line - a straight line that touches a curve at exactly one point only

Angles at points

R

Angles in a right-angle add to 90°



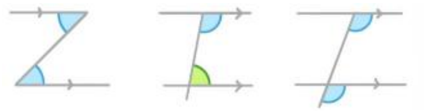
Angles on a straight line add to 180°

Angles round a point add to 360°

Vertically opposite angles are equal

Angles in parallel lines and shape

R



Alternate angles are equal | **Co-interior** angles add to 180° | **Corresponding** angles are equal

We can use parallel line rules to find missing angles when we have two parallel lines and a transversal.

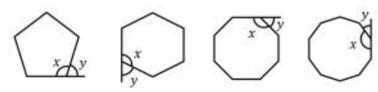
Exterior and interior angles of polygons

R

An interior angle (x) in inside the shape.

An exterior angle (y) is outside the shape.

An interior angle + adjacent exterior angle = 180°

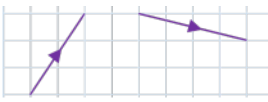


Interior angle = $\frac{(n-2) \times 180}{n}$ Exterior angle = $\frac{360}{n}$

Note: the above formulae are for **regular polygons** only

Prove geometric facts & solve problems with vectors

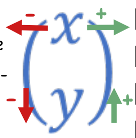
H



$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

A vector is a way of denoting a movement from one position to another.

The top value is the movement in the x-axis, whereas the bottom value is the movement in the y-axis. Negative values change the direction of the movement.



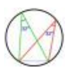
Review of circle theorems

H

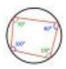
The angle subtended at the circumference is half the angle subtended at the centre

The angle in a semi-circle is a right-angle

Angles in the same segment are equal



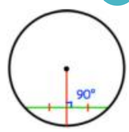
Opposite angles in a cyclic quadrilateral sum to 180°

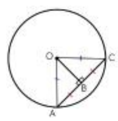


Angles between radius and chord

H

A radius that bisect a chord will meet the chord at 90°.





This image shows a general case of a radius meeting a chord. From this image, we can see that:

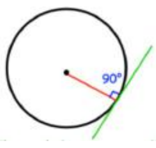
OC = OA AB = AC $\hat{ABO} = \hat{CBO} = 90°$

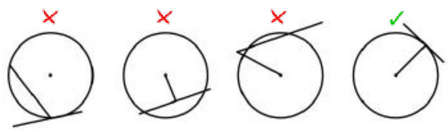
ABO and CBO are congruent triangles.

Angle between radius and tangent

H

The angle between a radius and a tangent line is 90. This is only true where they meet at the circumference of the circle.



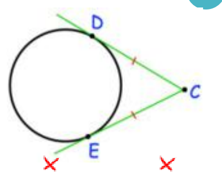


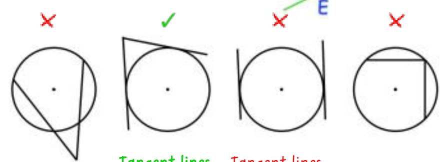
No radius shown | Radius meets chord | No tangent line shown | Radius meets tangent

Two tangents from a point

H

Two tangents from the same point will be equal in length.



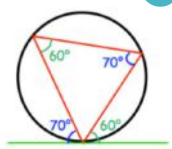


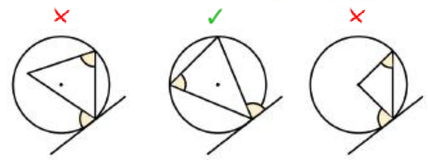
Not tangent lines | Tangent lines from same point | Tangent lines not from same point | Not tangent lines

Alternate segment theorem

H

The angle between the chord and the tangent is equal to the opposite angle inside a cyclic triangle.





Not a cyclic triangle | Opposite angle in a cyclic triangle | Not a cyclic triangle

YEAR 11 - UNIT 9
ALGEBRAIC REASONING

Sparx Codes: U486, U783, U498, U206, U530, U760, U757, U547, U582

What do I need to be able to do?

- Make and test conjectures about the generalisations that underlie patterns and relationships
- Use algebra to support and construct arguments and proofs
- Deduce expressions to calculate the nth term of linear and quadratic sequences
- Solve simultaneous equations in two variables algebraically and graphically
- Solve linear and quadratic inequalities in one or two variables
- Represent solutions to inequalities on a graph

Keywords

Nth term - a formula used to find any term in a sequence

Linear sequence - a sequence of numbers that increase/decrease by the same value each time

Quadratic sequence - a sequence of numbers that have a constant 2nd difference

Geometric sequence - a sequence of numbers that multiply by the same value each time

Fibonacci sequence - a sequence of numbers where each term is the sum of the previous two terms

Simultaneous equations - equations with two or more variables with the same value in each equation

Proof - logical arguments to show the truth of a mathematical statement

Simplify complex expressions

We can simplify by collecting like terms, multiplying terms, or dividing terms.

$3xy + 2yx = 5xy$ These are like terms even though the variables aren't in the same order.

$3a^2b - 5a^2b = -2a^2b$ These are like terms because the variables **and powers** are the same.

$3yz \times 5y = 15y^2z$ When multiplying, we don't need like terms. Just add the powers for any common variables.

$5q^3rs^5 \times -3qs^6 = -15q^4rs^{11}$ Remember: the "front number" is not a power, so we actually multiply them.

$\frac{12a^5b^3c^7}{3a^3b} = 4a^2b^2$ When dividing, we also don't need like terms. Just subtract the powers for any common variables.

Find the rule for the nth term of a linear sequence (R)

To find the nth term of a linear sequence, we look for two things:

the common difference

the 0th term (the previous term)

$\overset{-7}{6}$, 13, 20, 27, ...

+7 +7

nth term:

$7n + 6$

Find the rule for the nth term of a quadratic sequence (R) (H)

1, 12, 29, 52, ...

+11 +17 +23

+6 +6

Always divide the 2nd difference by 2

$\div 2 \rightarrow 3n^2$

Original: 1, 12, 29, ...

$3n^2$: 3, 12, 27, ...

Generate and subtract your new sequence from the original sequence

-2

-4 -2, 0, 2, ...

+2 +2

$3n^2 + 2n - 4$

Use rules for sequences

Linear/arithmetic sequence

5, 12, 19, 26, ...

+7 +7 +7

The terms add the same common difference each time

Quadratic sequence

3, 5, 10, 18, ...

+2 +5 +8

+3 +3

The terms add the same second difference each time

Geometric sequences

3, 30, 300, ...

X10 X10

The terms multiply by the same common ratio each time

Fibonacci sequences

1, 1, 2, 3, 5, ...

Each term is the sum of the previous two terms

Solve linear simultaneous equations (R)

Solve this pair of simultaneous equations:

$3a + 2b = 70$ $\quad b = a + 10$

Elimination method:	Substitution method:
b = a + 10 -> -a + b = 10	3a + 2b = 70
-a + b = 10 -> -2a + 2b = 20	3a + 2(a + 10) = 70
3a + 2b = 70	3a + 2a + 20 = 70
<u>-2a + 2b = 20</u>	5a + 20 = 70
5a = 50 -	5a = 50
a = 10	a = 10
b = a + 10 = 10 + 10 = 20	b = a + 10 = 10 + 10 = 20
a = 10 b = 20	a = 10 b = 20

Solve simultaneous equations with one quadratic (R) (H)

When simultaneous equations involve a quadratic, we cannot use the elimination method.

Solve: $x^2 + y^2 = 25$ & $y = x + 1$

$x^2 + (x + 1)^2 = 25$ -> $x^2 + x^2 + 2x + 1 = 25$

$2x^2 + 2x - 24 = 0$ -> $x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$ -> $y = -4 + 1 = -3$ $\quad y = 3 + 1 = 4$

$x = -4$ $\quad x = 3$ -> $x = -4, y = -3$ $\quad x = 3, y = 4$

Formal algebraic proof (H)

Here are some generalisations you can use to help with algebraic proofs.

n -> an integer

2n -> an even integer

2n - 1 -> an odd integer (2n + 1 also works)

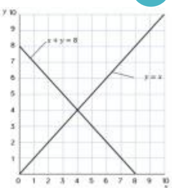
5n -> a multiple of 5 (5(____) also works)

n, n + 1, n + 2 -> three consecutive integers

Inequalities in two variables (H)

This graph shows two equations:

$x + y = 8$ & $y = x$



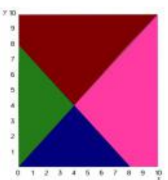
We can split the graph into four regions (shown below), then represent each region using inequalities.

$x + y \geq 8$ & $y \geq x$

$x + y \leq 8$ & $y \geq x$

$x + y \leq 8$ & $y \leq x$

$x + y \geq 8$ & $y \leq x$



Note: the inequalities are inclusive ($\leq$ or $\geq$), as the lines are solid, rather than dotted.

YEAR 11 - UNIT 10

TRANSFORMING & CONSTRUCTING

Sparx Codes: U799, U696, U196, U134, U766, U187, U787, U245, U820, U450, U598, U487, U455

What do I need to be able to do?

- Interpret and use fractional and negative scale factors for enlargements
- Describe the changes/invariance achieved by combinations of transformations
- Recognise, sketch, and interpret trigonometric graphs
- Sketch translations and reflections of graphs given a function

Keywords

Reflection - a shape as it would be seen in a mirror (the shape is flipped over the line)

Rotation - turning a shape around a centre point (spinning it around)

Translation - moving a shape without rotating/reflecting it (always produces congruent shapes)

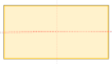
Enlargement - a transformation that proportionally changes the size of a shape

Invariant - a property that does not change after a given transformation

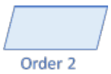
Locus/loci - a set of points that share a property/follow a given rule

Construction - an accurate drawing of a shape, angle, or line, often produced using compasses

Perform and describe symmetry and reflection (R)



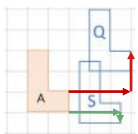
Line symmetry: a rectangle has 2 lines of symmetry, as it is the same on either side of each line.



Rotational symmetry: a parallelogram has rotational symmetry of order 2, as it fits into itself twice while performing a full rotation.

Perform and describe translations of shapes (R)

Translations are one of the four transformations. Translations involve movement of a shape only, so the resulting shape will be **congruent** with the original. We often use **vectors** to describe a translation.



A->Q is a translation of $\begin{pmatrix}3\\2\end{pmatrix}$

A->S is a translation of $\begin{pmatrix}2.5\\-0.5\end{pmatrix}$

Perform and describe enlargements of shapes (R)

An enlargement is a transformation that generally makes a shape larger or smaller.

A **positive scale factor** (>1) makes a shape **larger**.

A **fractional scale factor** (<1) makes a shape **smaller**.

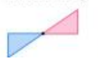
A **negative scale factor** (<0) enlarges the shape in the **opposite direction**.

Identify transformations of shapes (including series of transformations)

1. Reflection e.g.

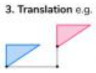
The shape has been **reflected** in the y-axis/in the line y = x/in the line y = 3x -1.

2. Rotations e.g.



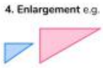
The shape has been **rotated** 90°/180°/270°, clockwise/anti-clockwise, around the centre (x, y).

3. Translation e.g.



The shape has been **translated** by the vector $\begin{pmatrix}x\\y\end{pmatrix}$.

4. Enlargement e.g.



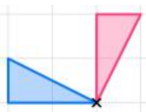
The shape has been **enlarged** by scale factor 3/0.25/-4 about the centre (x, y).

Identify invariant points and lines

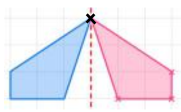
An invariant point/line is a point/line that doesn't change after a transformation (or series of transformations).

For **reflections**, invariant points are on the line.

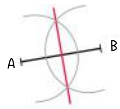
For **rotations and enlargements**, invariant points are on the centre.



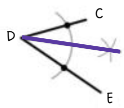
x Invariant point



Perform standard constructions using ruler and protractor or ruler and compasses



The perpendicular bisector cuts a line in half, at an angle of 90°. This is the perpendicular bisector of the line AB.

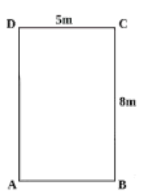


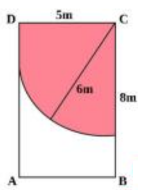
The angle bisector splits an angle in half. This angle bisector splits the angle CDE in half.

Solve loci problems

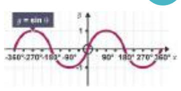
Loci problems often involve practical context. Try to produce a drawing that follows the rules provided.

A dog is tied to point C in the shed, with a rope of length 6m. Shade the area the dog can reach.

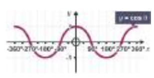




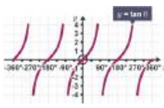
Understand and use trigonometrical graphs (H)



The graph for y = **sin(θ)** has a minimum y value of -1 and a maximum of 1. It has a period of 360°, meaning the graph repeats every 360°.

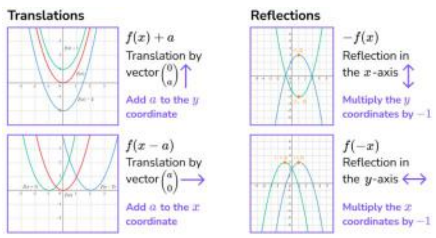


The graph for y = **cos(θ)** is identical to that of sin, just translated 90° to the left.



The graph of y = **tan(θ)** tends to ±∞ every 180°. It has a period of 180°.

Sketch and identify translations and reflections of the graph of a given function (H)



YEAR 11 - UNIT 11
LISTING & DESCRIBING

Sparx Codes: U104, U369, U476, U748, U699, U743, U277

What do I need to be able to do?

- Explore what can be inferred in statistical and probabilistic settings
- Calculate the probability of independent and dependent combined events
- Calculate and interpret conditional probabilities and expected frequencies
- Apply systematic listing and the product rule for counting

Keywords

Systematic - following a system that involves order, thoroughness, and regularity

Exhaustive - including all possible outcomes

Outcome - a single possible result of an experiment e.g. landing heads when flipping a coin

Event - one ore more outcomes of an experiment e.g. rolling an even number on a dice

With replacement - an experiment in which the object is put back into the set after the first choice

Product rule - multiply the number of outcomes from each selection to get the total outcomes

Union - the set made by combining two sets, can be read as OR

Intersection - the set made by picking the common elements from two sets, can be read as AND

Complement - all of the elements not in a given set, can be read as NOT

Plan view - view of an object from above "bird's eye view"

Front/side elevation - view of an object from the front/side

Hypothesis - a statement that can be tested to determine whether it is true or false

Interquartile range - the range of the middle half of a set of data

Outlier - a value that lies well outside the rest of the data set

Correlation - the link between two sets of data, often described as positive or negative

Work with organised lists

It is a good idea to work **systematically** to avoid duplicates and missing out options.

Sandwiches:
Cheese
Ham
Beef

Drinks:
Orange juice
Water

Cheese & OJ　　Cheese & Water

Ham & OJ　　Ham & Water

Beef & OJ　　Beef & Water

So there are **6 possible combinations**

Sample spaces and probability (R)

Sample space diagrams can be used to help organise a probability problem.

This sample space diagram shows the possible outcomes when rolling two dice and summing their rolls.

What is the probability of rolling exactly 7?

$P(7) = \frac{6}{36} = \frac{1}{6}$

Use the product rule for counting (H)

Rather than counting combinations, we can multiply the number of options for each choice.

Sandwiches:
Cheese
Ham
Beef

Drinks:
Orange juice
Water

3 sandwiches X 2 drinks = **6 combinations**

This is good for saving time and checking answers, but it doesn't tell us the different options.

Complete and use Venn diagrams (R)

Intersection ∩: all elements that are in both sets. So **A ∩ B** means elements in **A AND B**.

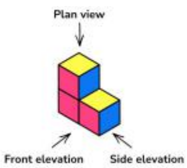
Union U: all elements from either set. So **A U B** means elements in **A OR B**.

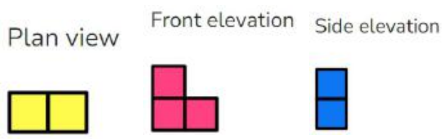
Complement ': all elements not in the given set. So **A'** means **NOT A**.

Universal set ξ: a set containing all elements.

Construct and interpret plans and elevations (R)

The **plan view** of a shape is the view from above. The **front/side elevations** are the view from the front/side.





Use data to compare distributions (R)

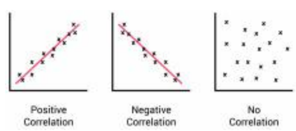
We typically compare data using either an **average**, or a **measure of spread**.

An **average** (the mean, median, or mode) gives us a value that represents a group. So we could say one group was slower on average, younger on average, or taller on average.

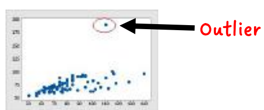
A **measure of spread** (range or interquartile range) tells us how spread out our data is. So we could say one set is more consistent.

Interpreting scatter diagrams (R)

We can recognise **correlation** from a scatter graph.



We can also look for outliers. **Outliers** are points that don't fit the pattern of the data.



YEAR 11 - UNIT 12

SHOW THAT

Sparx Codes: U550, U689, U582, U471, U887, U560, U903, U564, U781, U866

What do I need to be able to do?

- Use algebra to support and construct arguments and proofs
- Apply the concepts of congruence and similarity
- Make connections between areas of maths
- Change between recurring decimals and fractions
- Perform operations with vectors, use them to construct arguments and proofs

Keywords

Proof - logical arguments to show the truth of a mathematical statement

Counter-example - an example that disproves a statement (shows that it is false)

Quartile - values that divide a list of numbers into quarters

Parallel - parallel lines have the same gradient, so they never meet

Collinear - points that lie on a single straight line

Condition - necessary criteria for a statement, theorem, or proof to be true

Congruent - two shapes are congruent if they have exactly the same lengths and angles

Show that - number

Sum - the result of **adding** numbers

Difference - the result of **subtracting** numbers

Product - the result of **multiplying** numbers

Quotient - the result of **dividing** numbers

To convert a recurring decimal to a fraction, follow the steps below.

$$0.\dot{4}\dot{5} = x \qquad 45.\dot{4}\dot{5} = 100x$$

$$\begin{aligned} 45.\dot{4}\dot{5} &= 100x \\ -\ 0.\dot{4}\dot{5} &= x \\ 45 &= 99x \end{aligned}$$

$$\begin{aligned} 45 &= 99x \\ \tfrac{45}{99} &= x \end{aligned}$$

Show that - algebra

Term - a single number, variable, or variable and number multiplied together (e.g. 7, a, x, 3x, 4.5y)

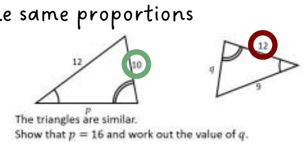
Expression - multiple terms with operations between them, but no equals sign or inequalities

Equation - multiple terms with operations between them and an equals sign

Identity - an equation that is true for all values of the variables, denoted by the $\equiv$ symbol

Show that - shape

Similar - two shapes are similar if they have the same proportions

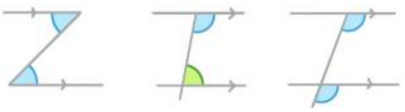


12 ÷ 9 = 1.333... <- scale factor

16 ÷ 1.333... = 12 <- this is correct, so p = 16

10 ÷ 1.333... = 7.5 <- so q = 7.5

Show that - angles



Alternate angles are equal | **Co-interior** angles add to 180° | **Corresponding** angles are equal

	0°	30°	45°	60°	90°
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	*undefined*

Show that - data

We typically compare data using either an **average**, or a **measure of spread**.

Mean - add all the values, then divide by the number of values

Median - order the values and find the middle

Mode - the most common value

Range - largest value minus the smallest value

IQR - upper quartile minus the lower quartile

Show that - vectors

H

For **addition and subtraction**, just add or subtract the top and bottom numbers.

$$\begin{pmatrix}2\\5\end{pmatrix} + \begin{pmatrix}-1\\7\end{pmatrix} = \begin{pmatrix}1\\12\end{pmatrix} \qquad \begin{pmatrix}11\\3\end{pmatrix} - \begin{pmatrix}5\\-7\end{pmatrix} = \begin{pmatrix}6\\10\end{pmatrix}$$

$$4\begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}12\\-20\end{pmatrix}$$

For **scalar multiplication**, multiply the scalar by the top and the bottom number.

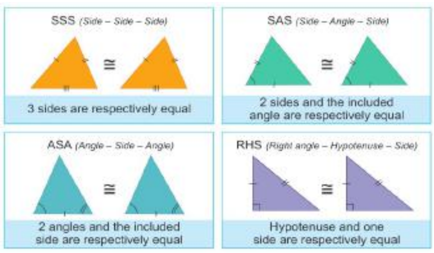
Vectors are **parallel** if they are a scalar multiple of one another.

$$-2\begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}-8\\-2\end{pmatrix}$$

So $\begin{pmatrix}4\\1\end{pmatrix}$ *and* $\begin{pmatrix}-8\\-2\end{pmatrix}$ *are parallel.*

Show that - congruent triangles

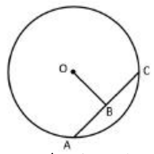
Triangles are **congruent** if all sides and angles are equal. If any of the following criteria are met, we can guarantee congruency.



Formal proof with congruent triangles

H

The diagram shows a chord AC drawn through a circle centre O.
The midpoint of AC is B.
Show that triangles OAB and OCB are congruent



OC and OA are equal, as they are both radii.

The radius bisects the chord at 90°, so OBA and OBC must be right-angles.

OC is a common side in both triangles.

$\therefore$ OAB and OCB are congruent due to RHS.

Revision

Congratulations - you've finished the course content! Here are some suggestions for revision:

mathsgenie - revision by topic & mini tests

corbettmaths - revision by topic & 5-a-days

onmaths - self marking online practice papers

sparxmaths - revision by topic with video support